

Endogenous Technological Change Along the Demographic Transition^{*}

Markus Pettersson[†]

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Abstract

Does population ageing hurt output per capita? Standard life-cycle models with exogenous growth that emphasise two opposing forces, capital deepening versus declining employment rates, predict yes. Using a quantitative overlapping generations model with realistic demographics and R&D-driven endogenous growth, this paper challenges that prediction through a third channel: that ageing-induced increases in household savings also finance R&D investment and therefore generate technological progress. Calibrated to the United States, the model implies that the demographic transition between 1950 and 2100 increases annual per-capita growth by 0.33 percentage points until 2000 and by 0.16 percentage points overall, thereby accounting for 15 to 20 percent of observed US growth and cumulating into a 27.5 percent permanently higher level of output per capita by 2100. The key mechanism is endogenous technological change, whose growth contribution triples that of capital deepening; shutting down this channel eliminates the positive impact altogether.

Keywords: demographic transition, endogenous growth, OLG model.

JEL Codes: E17, E25, J11, O30, O40.

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[†] Department of Economics, Stockholm University. Email: markus.pettersson@su.se.

1 Introduction

In 1950 there were 14 people aged 65 and above for every 100 people aged 20 to 64 in the United States. This number has risen steadily since and is projected to quadruple by the end of the twenty-first century in response to rising life expectancy and low fertility rates. Similar trends apply to high-income countries worldwide, with the rest of the world trailing not far behind. This dramatic shift raises serious macroeconomic concerns, and an important aspect underlying many of them is the impact of demographic change on national income.

How, then, does population ageing affect output per capita? Conventional wisdom based on life-cycle reasoning suggests two opposing channels. First, ageing reduces the fraction of workers in the population, thereby lowering output per capita. Second, rising life expectancy increases saving rates in anticipation of longer retirement, which spurs investment. The resulting capital deepening increases output per capita. Even setting aside the severe identification issues between growth and demographics—let alone any underlying mechanisms—it therefore comes as no surprise that the empirical evidence on the net effect remains inconclusive.¹ Quantitative theory offers a solution to this ambiguity, and life-cycle models calibrated to advanced economies typically predict the employment effect to dominate. Krueger and Ludwig (2007) and Ludwig, Schelkle and Vogel (2012) for instance find that demographic change during the twenty-first century generates cumulative declines in output per capita on the order of 10 percent.

While subsequent research has added several mechanisms to these conventional two, one of the most natural growth channels remains largely unexplored in this context: higher saving may also finance investment in research and development (R&D), whose innovations raise output through technological progress. This mechanism matters because technology, unlike capital, is nonrival. An innovation can be used simultaneously by any number of workers at no additional cost and therefore raises everyone's productivity. Capital, by contrast, improves productivity only insofar as it increases capital per effective worker. Technological change can therefore generate stronger productivity effects than capital deepening. Suggestive evidence also points in this direction: Figure 1 shows that across a 1970–2020 panel of OECD countries, a one-year increase in adult life expectancy corresponds to a 0.70 percentage point higher saving rate and a 0.20 percentage point higher investment rate in intellectual property products, conditional on relative prices and dependency ratios.²

This paper quantifies this R&D channel in a calibrated general equilibrium model and shows that it reverses both the sign and magnitude of the conventional prediction. The model combines endogenous growth à la Romer (1990) with a rich overlapping-generations (OLG) structure in the Auerbach and Kotlikoff (1987) tradition. On the production side, monopolistically competitive intermediate producers and an R&D sector generate technological progress by expanding the variety of intermediate goods. On the household side, life-cycle agents consume, save, and make labour-supply decisions—both hours worked and age of retirement—subject to changes in household size, mortality and income risk, borrowing constraints, progressive income taxation, and bequest motives. Savings are either invested in capital or used to finance R&D in exchange for ownership stakes in the new firms it generates, with arbitrage equalising their returns in equilibrium, and this financing margin links household behaviour to technological progress.

¹ See for instance Maestas, Mullen and Powell (2023) and the variety of estimates from the debate surrounding Acemoglu and Johnson (2007) by Lorentzen, McMillan and Wacziarg (2008), Aghion, Howitt and Martin (2011), Cervellati and Sunde (2011), Bloom, Canning and Fink (2014), and Hansen and Lønstrup (2015).

² Using a panel of OECD countries from 1960 to 2011, Gehring and Prettnner (2017) find that a 10 percent decrease in the crude death rate leads to a 1 percent increase in total factor productivity. See also Bloom, Canning and Graham (2003) and Bloom *et al.* (2007) for empirical support that higher life expectancy raises saving rates.

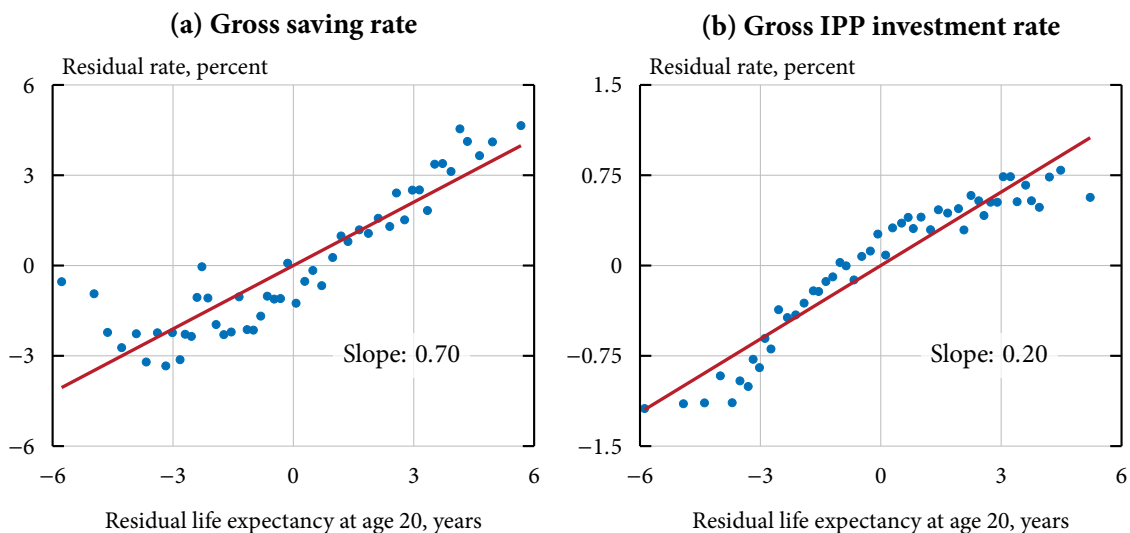


Figure 1. Partial correlations of savings, investment, and life expectancy, 1970–2020.

Notes. Data from the OECD and United Nations (2024). The figures depict demeaned binscatters over OECD members after partialling out country fixed effects, relative prices, and dependency rates (0–14 and 65+ population shares) following the method proposed by Cattaneo *et al.* (2024). The red line shows the OLS fit. IPP refers to intellectual property products (a superset of formal R&D).

The model exhibits semi-endogenous growth as in Jones (1995), Kortum (1997), Segerstrom (1998), and Peters and Walsh (2026), but its rich life-cycle structure departs from the representative-agent and perpetual-youth settings typically found in this literature. This contribution is central to the question at hand. In semi-endogenous growth theory, long-run economic growth is tied to population growth, which makes demographic analyses straightforward as long as current trends truly reflect shifts in steady-state population growth. However, since World War II, demographic change in the advanced economies of North America and Western Europe primarily reflects two temporary forces: an initial surge in fertility (the baby boom) and steadily rising survival rates beyond age 50, well past childbearing years.³ Neither affects long-run population growth provided there exists an upper bound on human lifespans. Instead, these demographic changes generate temporary growth effects that permanently affect the *level* of income, but such level effects cannot be properly examined through the lens of steady-state growth in a representative-agent or perpetual-youth model.

To shed light on these growth effects and complement pure long-run analyses, this paper examines demographic changes around a fixed long-run trend, which allows the key population trends to be treated as transitory.⁴ After illustrating the mechanisms in an analytically tractable three-generation model, I develop and calibrate the full quantitative model and simulate the equilibrium path induced by the demographic transition between 1950 and 2100. The analysis focuses on the United States, a large frontier economy where the endogenous growth framework arguably provides a better description of technological change than in smaller economies that primarily adopt foreign technology. The United States also serves as a useful demographic benchmark, as its ageing trajectory lies between that of the fastest-ageing advanced economies and younger non-OECD countries.

³ Figure F.1 and Figure F.2 in Online Appendix F document these two facts. Besides the baby boom, fertility remains stable from the mid-1930s onward. Likewise, mortality during the first 50 years of life is largely unchanged after 1950.

⁴ In doing so, I abstract from the possibility studied by Jones (2022a) that persistently low fertility could lead to a declining population—an important but ultimately separate issue.

Contrary to what a standard long-run analysis suggests, I find that the demographic transition is a net positive. On average, output per capita growth increases by 0.33 percentage points per year between 1950 and 2000 and by 0.16 percentage points per year overall. The former corresponds to roughly 15–20 percent of observed US growth over the same period, thereby making demographics as important to US post-war growth as rising educational attainment according to estimates by Fernald and Jones (2014). Counterfactual simulations reveal that the baby boom and rising life expectancy after the age of 50 explain around half of these results. Changes in the age structure driven by the nonstationary population already present in 1950 and general increases in population size explain the remainder. As far as the impact on output per capita is concerned, these findings leave little reason to worry about current demographic change.

The positive effect dissipates after the turn of the millennium when the baby boom retires, but the growth impact does not turn negative on average during the twenty-first century. Yet, the shift when this happens is sizeable: the growth rate declines by 0.53 percentage points between 1995 and 2030. Population ageing therefore exerts significant downward pressure on contemporary growth rates, just as Eggertsson, Lancastre and Summers (2019) and Jones (2023) argue in the context of secular stagnation. It is important to stress though that this decline is *not* a result of demographic change being inherently detrimental to economic output, but rather because of a reversion from the above-average growth that it induced in the first place.

At the core of these results lies technological change, which contributes three to four times more to output growth than capital deepening. This difference reflects the added productivity gains obtained from technology being nonrival. Together these two channels more than offset the negative impact of a growing share of retired households. Capital deepening alone cannot accomplish this: treating technology as exogenous (an immediate special case of the benchmark model) eliminates the positive effect and leads to a 12.2 percent cumulative decline in output per capita between 1950 and 2100. This decline is quantitatively similar to Krueger and Ludwig (2007) and contrasts with a 27.5 percent increase in the baseline scenario, thus underlining that whether the demographic transition raises or lowers output per capita depends crucially on whether or not we account for its impact on technological progress.

The idea that the demographic structure interacts with R&D-based growth is well established, dating back at least to Malmberg (1994) and further emphasised by, among others, Prettnner (2013), Gehringer and Prettnner (2017), Prettnner and Trimborn (2017), Baldanzi, Prettnner and Tscheuschner (2019), Aksoy *et al.* (2019), and Basso and Jimeno (2021).⁵ This paper differs from this literature along two dimensions. First, existing contributions typically use analytically tractable models, often based on perpetual-youth frameworks in which mortality and retirement (if present) are independent of age. While useful for isolating mechanisms, these models impose strong restrictions on life-cycle behaviour and lack the flexibility to match empirical age distributions, which limits their suitability for quantitative statements relative to full-fledged OLG models, as explained by Carvalho, Ferrero and Nechio (2016) and Gagnon, Johannsen and López-Salido (2021). Second, this literature interprets population ageing primarily through changes in steady-state population growth. By contrast, this analysis treats demographic transitions as inherently non-stationary phenomena and focuses on their transitional implications. As argued above, this distinction matters, since it can overturn standard predictions about the impact on output per capita.

⁵ Cutler *et al.* (1990) also suggest that ageing generates technological change via automation, which Acemoglu and Restrepo (2022) and Abelianky and Prettnner (2023) reiterate with evidence on two outcomes: robot adoption and the share of patents related to automation. Both margins are consistent with the mechanisms here: the former reflects capital deepening, while the latter concerns the direction, not the volume, of innovation.

This paper also adds to the broader quantitative macroeconomic literature, where R&D-based growth is largely confined to business cycle analyses of a representative household.⁶ The extensive OLG literature on demographic change instead focuses primarily on fiscal policy, international capital flows, and asset returns.⁷ Studies such as Krueger and Ludwig (2007) and Cooley and Henriksen (2018) that examine growth in this context typically concentrate on the trade-off between capital deepening and a shrinking workforce, although several papers do incorporate additional margins such as human capital accumulation (Ludwig, Schelkle and Vogel, 2012; Vandenbroucke, 2021), automation (Heer and Irmen, 2014; Benzell *et al.*, 2021), and public investment (Gonzalez-Eiras and Niepelt, 2012). These additional mechanisms similarly operate through investment in productivity-enhancing activities, thereby naturally complementing the R&D channel emphasised here. By isolating this channel within a quantitative OLG framework, this paper marks an initial step toward integrating R&D-driven growth into this body of analysis.

2 A Simple Model to Understand the Mechanisms

Permanent level changes and transitional growth necessarily go hand in hand. To build intuition for the transition dynamics in the quantitative model, I therefore begin with a simplified environment that admits an analytical solution. Consider the Solow-esque framework in Table 1, in which households exogenously save a fixed fraction s of income and supply labour L_t , which grows with population at rate n . Savings are allocated between next-period capital K_{t+1} and R&D investment Q_t , which together with the existing technology stock Z_t generates new technologies through an R&D production function similar to Jones (1995). Output Y_t is determined by Cobb-Douglas production with capital, technology, and labour as inputs. For simplicity, capital and technology fully depreciate each period, and a fixed fraction $\bar{\rho}$ of savings is invested in capital. Under full depreciation, a fixed allocation is also the competitive equilibrium allocation in richer frameworks à la Romer (1990), but imposing it directly simplifies the exposition.

We are interested in how the demographic structure affects output per capita, which by the Cobb-Douglas production function can be written

$$y_t \equiv \frac{Y_t}{N_t} = Z_t \left(\frac{K_t}{Y_t} \right)^{\frac{\alpha}{1-\alpha}} \frac{L_t}{N_t}, \quad (1)$$

where N_t is the size of the population. Output per capita is thus proportional to the stock of technology (total factor productivity, TFP), a capital intensity (captured by the capital-output ratio), and the employment rate. Recalling that demographic change after 1950 is characterised by two salient features—the baby boom and increasing life expectancy after the age of 50—the focus below is to highlight how these two affect the components of Equation (1).

2.1 Components of Output Per Capita in Steady State

Table 1 outlines a semi-endogenous growth model in which the components of Equation (1) can be solved explicitly in steady state—an equilibrium where all variables grow at constant (possibly zero) rates. First, dividing (S2) by Z_t yields the gross TFP growth rate $1 + g_{Zt} \equiv \frac{Z_{t+1}}{Z_t} = Q_t^\lambda Z_t^{\phi-1}$. In steady state, the right-hand

⁶ Examples include Comin and Gertler (2006), Nuño (2011), Benigno and Fornaro (2018), Anzoategui *et al.* (2019), Bianchi, Kung and Morales (2019), and Okada (2022).

⁷ Besides papers mentioned elsewhere, a far from exhaustive list includes De Nardi, İmrohoroglu and Sargent (1999), Storesletten (2000), Fehr, Jokisch and Kotlikoff (2004), Börsch-Supan, Ludwig and Winter (2006), Domeij and Flodén (2006b), Attanasio, Kitao and Violante (2007), Kotlikoff, Smetters and Walliser (2007), İmrohoroglu and Kitao (2012), Kitao (2014), Gagnon, Johannsen and López-Salido (2021), and Auclert *et al.* (2025).

Table 1. Summary of the simple model.

Output	$Y_t = K_t^\alpha (Z_t L_t)^{1-\alpha},$	$0 < \alpha < 1$	(S1)
R&D output	$Z_{t+1} = Q_t^\lambda Z_t^\phi,$	$0 < \lambda \leq 1, \phi < 1$	(S2)
Asset market	$sr Y_t = K_{t+1} + Q_t$		(S3)
Allocation	$Q_t = (1 - \bar{\rho}) sr Y_t,$	$0 < \bar{\rho} < 1$	(S4)
Population growth	$L_{t+1} = (1 + n) L_t$		(S5)

side must remain constant while R&D investment must grow at the rate of total output. It follows that steady-state TFP growth satisfies

$$1 + g_Z = (1 + n)^\gamma, \quad \text{where} \quad \gamma \equiv \frac{\lambda}{1 - \phi - \lambda}, \quad (2)$$

and this is a function of exogenous parameters, hence the “semi-endogenous” modifier. Combining Equation (2) with (S3) and (S4) gives the capital-output ratio:

$$\frac{K_t}{Y_t} = \frac{\bar{\rho} sr}{(1 + n)^{\frac{\gamma(1-\phi)}{\lambda}}}. \quad (3)$$

Like the Solow model, the capital intensity is thus the share of output allocated to capital investment divided by the aggregate growth rate. Equations (2) and (3) can then be used with those in Table 1 to solve for the stock of TFP:

$$Z_t = \left[\frac{1 - \bar{\rho}}{\bar{\rho}} \left(\frac{\bar{\rho} sr}{(1 + n)^{\frac{\gamma(1-\phi)}{\lambda}}} \right)^{\frac{1}{1-\alpha}} \right]^\gamma L_t^\gamma. \quad (4)$$

Equation (4) writes TFP as the product of two factors. The first is an R&D intensity, which is determined by the saving rate and thus reflects household behaviour and composition. The second, L_t^γ , is the scale effect on output per capita present in all semi-endogenous growth models (Jones, 2005). Specifically, for given household behaviour, a larger population increases aggregate investment, and this raises the stock of TFP. But technology is nonrival; once discovered, it can be used simultaneously by everyone at no additional cost. Output per capita therefore depends on total TFP rather than TFP per capita, as seen in Equation (1). Consequently, output per capita increases simply by virtue of a larger population.

At this point, let's add households to endogenise the employment and saving rates. Consider a life-cycle model with young, middle-aged, and old households (indexed by $j = 1, 2, 3$, with cohort size N_{jt}). Young and middle-aged households supply labour inelastically, while old households are retired. If the size of the young cohort grows at rate $1 + n$ each period, and a j -year-old survives to age $j + 1$ with probability s_j , then the steady-state employment rate is

$$\frac{L_t}{N_t} = \frac{N_{1t} + N_{2t}}{N_{1t} + N_{2t} + N_{3t}} = \frac{1 + \frac{s_1}{1+n}}{1 + \frac{s_1}{1+n} \left(1 + \frac{s_2}{1+n} \right)}. \quad (5)$$

For simplicity, suppose households have logarithmic preferences over consumption c , that annuity markets are present, and that young households consume their entire income. Households then maximise expected

lifetime utility $\log(c_1) + s_1\beta \log(c_2) + s_1s_2\beta^2 \log(c_3)$ subject to the budget constraints $c_1 = w$, $c_2 + a = w$, and $c_3 = \frac{1+r}{s_2} a$, where β , a , r , and w denote the discount factor, household assets, the interest rate, and the wage rate, respectively. The solution to the household problem is characterised by the savings policy $a_{t+1} = \frac{s_2\beta}{1+s_2\beta} w_t$. The aggregate steady-state saving rate can then be written

$$sr = \frac{a_{t+1}N_{2t}}{Y_t} = \frac{w_t L_t}{Y_t} \cdot \frac{s_3\beta}{1+s_3\beta} \cdot \frac{\frac{s_2}{1+n}}{1+\frac{s_2}{1+n}}, \quad (6)$$

which is a product of the labour share of national income, the share of labour income allocated to savings, and the share of middle-aged workers (i.e., savers) in the labour force. Note that the labour share of income is constant given the Cobb-Douglas production function with constant exponents.

2.2 Population Ageing via Increasing Survival Rates

Consider now an ageing of the population through increases in the survival rates s_1 and s_2 . Think of the former as representing the increase in survival rates for ages 50–64 observed in the data, while the latter represents the similar increase observed for ages 65 and above. The higher survival rates naturally push down the employment rate through a mechanical increase in the old-age dependency ratio, as seen in Equation (5). This reduces output per capita by (1). Meanwhile, Equations (3), (4) and (6) highlight a counteracting force: higher survival rates raises the saving rate, which in turn generates larger capital and R&D intensities, the latter of which improves TFP. Additionally, an increase in the middle-age survival rate s_1 also raises the size of the labour force relative to its previous trend, thereby increasing TFP further through the scale effect present in (4).

The intuition for the effects on the capital and R&D intensities is straightforward. A higher likelihood of middle-age survival raises the share of households that save, while a higher likelihood of old-age survival increases the life-cycle savings motive, so households save more intensively. The extra savings are allocated proportionately to capital and R&D investment according to (S4). The capital stock, however, increases disproportionately more than TFP because the production of capital is linear in investments whereas the R&D process features decreasing returns to scale: a necessary parameter restriction for a steady state to exist is $\gamma > 0$, which holds for feasible λ and ϕ if and only if $\lambda + \phi < 1$. Hence, both K_t/Y_t and Z_t rise.

2.3 A Baby Boom via Temporarily Increasing Fertility Rates

Next consider a temporary increase in the population growth rate $1 + n$, similar to a baby boom. Since population growth eventually returns to its original level, this leaves the long-run age structure unchanged. Thus, neither the employment rate nor the saving rate (and, by extension, the capital and R&D intensities) is affected. The only long-run impact is via the scale effect in (4): a temporary fertility boom permanently raises the population size, which leads to higher TFP and subsequently higher output per capita. This is exactly what a standard semi-endogenous growth model with a representative household predicts.

Equations (3) to (6) nevertheless provide guidance about the transition dynamics beyond the scale effect for this scenario. The saving rate is low when the share of middle-aged households is low, and the employment rate is high when the share of old households is low. The initial boost in the share of young households therefore generates positive growth via the employment rate and reduced growth via the capital and R&D intensities. As this generation reaches middle age and their children enter the labour force, the positive impact on the employment rate remains at the same time as the saving rate recovers, thus reverting the

Table 2. Comparative statics of the simple model.

Change	Duration	Output per capita	Technology		Capital intensity	Employment rate
			R&D intensity	Scale		
Mid-age mortality: $s_1 \uparrow$	Permanent	Ambiguous	+	+	+	–
Old-age mortality: $s_2 \uparrow$	Permanent	Ambiguous	+		+	–
Fertility: $1 + n \uparrow$	Temporary	+		+		

Notes. Increases denoted by plus signs, decreases denoted by minus signs.

effect on the capital and R&D intensities. Finally, when this generation reaches retirement, the employment rate declines back to its initial level, which completes the transition. These predictions will be useful to understand the quantitative results in later sections.

2.4 Taking Stock

The steady-state predictions are summarised in Table 2. Overall, the demographic transition lowers the employment rate through the old-age dependency ratio, raises the capital intensity via an increased saving rate, and generates technological change through both the saving rate and the scale effect associated with the nonrivalry of technology. Yet, the relative strengths of these channels and the net effect on transitory growth are ambiguous. This motivates the need for a quantitative treatment.

3 A Quantitative OLG Model With Endogenous Growth

The quantitative model is populated by overlapping generations of households, production firms, R&D firms, and a government. Time is discrete and a period is one year. Exogenous change in the demographic structure constitutes the driving force of the model.⁸ I adopt a closed-economy framework as a standard and conservative benchmark for analysing these changes. While international capital flows attenuate the link between domestic saving and investment, and recent work emphasises that demographics drive global imbalances (Bárány, Coeurdacier and Guibaud, 2023; Auclert *et al.*, 2025), Krueger and Ludwig (2007) find only modest differences in US GDP per capita between closed- and open-economy scenarios in similar exogenous-growth settings. Moreover, since population ageing is a global phenomenon and technological progress is nonrival, openness would likely strengthen rather than weaken the role of R&D.

The model concentrates its richness on the household side, since the paper’s mechanisms operate through demographic effects on life-cycle behaviour. Several empirically relevant household features are therefore added relative to the simple framework in Section 2: a realistic age distribution matched to demographic data; endogenous retirement, so that rising life expectancy does not translate one-for-one into employment declines; intensive margins of labour supply; changes in household size as dependants are born, grow up, and eventually move out; idiosyncratic income risk, borrowing constraints, and warm-glow bequests, which add savings motives beyond the standard life-cycle motive; and progressive income taxation alongside a pay-as-you-go social security system, which generates important fiscal adjustments as populations age. These features are not meant to introduce competing mechanisms but rather to discipline the household

⁸ I abstract from technological change feeding back into demographic variables (birth control, treatment of diseases, and so on), as recommended by Lee (2016); for a complementary multi-sector OLG analysis in which rising life expectancy and directed technical change towards the health sector are jointly endogenous, see Huetsch, Krueger and Ludwig (2026).

responses that feed into production and R&D when demographics change.⁹ The rest of the model remains close to the idea-based growth literature: the production sector consists of a perfectly competitive final-good firm and monopolistic intermediate-good firms, and technology grows endogenously through the entry of new intermediate firms created by innovations from R&D.¹⁰

3.1 Demographics and Household Composition

In period t , the economy consists of $J + 1$ generations of sizes $N_{0t}, \dots, N_{Jt}$, and the total population is $N_t = \sum_{j=0}^J N_{jt}$. From a given initial population, the demographic structure in subsequent periods is pinned down recursively by age and time specific fertility rates f_{jt} , survival rates s_{jt} , and net migration rates m_{jt} via

$$N_{0,t+1} = \sum_{j=0}^J f_{jt} N_{jt} \quad \text{and} \quad N_{j+1,t+1} = (s_{jt} + m_{jt}) N_{jt}, \quad (7)$$

where $s_{Jt} = m_{Jt} = 0$ holds for all t , implying that individuals die with certainty after age J . Migration is included because observed population dynamics cannot be matched without it, but otherwise it plays no independent role in the economic analysis. I thus assume that migrants bring their accumulated wealth when they move and are economically identical to non-migrants. This simplification eliminates the need to distinguish between the two groups in the economic model.

Individuals are children for the first ι years of their lives, after which they form a household of their own and become economically active. A household consists of an adult head and their dependants, including both children and grandchildren of underaged parents. For an adult of age j , the number of dependants of age $i < \iota$ is given by

$$N_{ijt}^d = f_{j-1-i,t-1-i} \frac{N_{j-1-i,t-1-i}}{N_{0,t-i}} \frac{N_{it}}{N_{jt}} + \sum_{k=i+1}^{\iota-1} N_{kjt}^d N_{ikt}^d. \quad (8)$$

The first term is the number of children. It is obtained by taking the fraction of the i -year-olds born to parents who are now age j , multiplying it by the current population of i -year-olds, and distributing that total evenly across the current j -year-old population. The second term is the number of grandchildren, where the number of i -year-old dependants of an underaged parent is given analogously to the first term on the right-hand side of (8).

3.2 Household Endowments and Preferences

Individuals are endowed with one unit of time and enter adulthood as members of the labour force with no assets. Workers supply $\ell_j = \varepsilon_j \eta h_j$ efficiency units of labour, where h_j is hours worked, ε_j is deterministic age-dependent productivity, and η is an idiosyncratic productivity component that evolves according to a time-invariant Markov chain. Initial productivity at age ι is distributed according to the chain's invariant distribution. Households partially self-insure against idiosyncratic risk through asset holdings a subject to a borrowing constraint. These holdings include both capital and equity claims on R&D-generated firms; no-arbitrage equalises equilibrium returns across the two, so the household problem reduces to a choice of total assets, with the portfolio composition pinned down by market clearing. Additionally, there are no annuity markets to insure against mortality risk. Instead, the government redistributes assets of households that die prematurely as lump-sum bequest transfers tr to surviving households. Suppressing

⁹ [Online Appendix E](#) confirms that shutting down several of these aspects in turn leaves the qualitative conclusions intact.

¹⁰ The aggregate mechanisms emphasised in this paper are present in any standard endogenous growth framework, so choosing between models with expanding varieties (Romer, 1990), quality ladders (Aghion and Howitt, 1992; Grossman and Helpman, 1991), or richer firm dynamics (Klette and Kortum, 2004) is largely inconsequential for the question studied here.

time subscripts in this subsection, the household budget constraint at age j is then

$$a_{j+1} + (1 + \tau^c)c_j = (1 + r(1 - \tau^k))a_j + (1 - \tau^w(w\ell_j) - \tau^b)w\ell_j + tr + b(R_j), \quad (9)$$

where c_j is household consumption, r is the rate of return on savings, w is the wage rate, and $\tau^c, \tau^k, \tau^w, \tau^b$ denote tax rates, with τ^w potentially varying with labour income. The term $b(R_j)$ denotes pension benefits (specified below), which depend on retirement status and retirement age, both summarized by R_j .

A household head of age j values their own consumption c_j^a , their leisure time $1 - h_j$, the consumption c_{ij}^d per i -year-old dependant, and the warm glow of knowing a bequest is left in case of future death. Starting with consumption, its flow utility is generated by

$$u_c \left(c_j^a, \{c_{ij}^d\}_{i=0}^{\iota-1} \right) = \frac{(c_j^a)^{1-\sigma} - 1}{1 - \sigma} + \sum_{i=0}^{\iota-1} \omega_i N_{ij}^d \frac{(c_{ij}^d)^{1-\sigma} - 1}{1 - \sigma}, \quad (10)$$

where $1/\sigma$ is the intertemporal elasticity of substitution for consumption and ω_i is the utility weight of i -year-old dependants. Households benefit from economies of scale in the sense that total household consumption satisfies the constant elasticity of transformation function

$$c_j = \left[(c_j^a)^\zeta + \sum_{i=0}^{\iota-1} N_{ij}^d (c_{ij}^d)^\zeta \right]^{\frac{1}{\zeta}}, \quad \zeta \geq 1, \quad (11)$$

in which the parameter ζ captures the degree of scale. If $\zeta = 1$, there are no scale benefits and total household consumption is just the sum of the individual consumption levels. Equations (10) and (11) admit an equivalent utility function expressed in terms of total household consumption. Taking household consumption as given and maximising (10) subject to (11) with respect to individual allocations, we obtain $c_{ij}^d = \omega_i^{\frac{1}{\zeta-(1-\sigma)}} c_j^a$. This condition combined with (10) and (11) generates a utility function of the form

$$u_c(c_j) = \Omega_{jc} \frac{c_j^{1-\sigma} - 1}{1 - \sigma}, \quad \text{where} \quad \Omega_{jc} \equiv \left[1 + \sum_{i=0}^{\iota-1} \omega_i^{\frac{\zeta}{\zeta-(1-\sigma)}} N_{ij}^d \right]^{\frac{\zeta-(1-\sigma)}{\zeta}} \quad (12)$$

is an age and time specific household size taste shifter. Specifically, substituting $c_{ij}^d = \omega_i^{\frac{1}{\zeta-(1-\sigma)}} c_j^a$ into Equation (11) reveals that $c_j = \Omega_{jc}^{\frac{1}{\zeta-(1-\sigma)}} c_j^a$, so $\Omega_{jc}^{\frac{1}{\zeta-(1-\sigma)}}$ measures the number of adult equivalents in a household and $\omega_i^{\frac{\zeta}{\zeta-(1-\sigma)}}$ is the equivalence weight of an i -year-old dependant. In terms of household consumption, leisure, and bequests, overall utility is subsequently defined as

$$u(c_j, h_j, a_{j+1}^b) = \Omega_{jc} \frac{c_j^{1-\sigma} - 1}{1 - \sigma} - \psi \frac{h_j^{1+1/\theta}}{1 + 1/\theta} + (1 - s_j) \Omega_a \frac{(a_{j+1}^b)^{1-\sigma} - 1}{1 - \sigma}, \quad (13)$$

where θ is the Frisch elasticity of labour supply, ψ determines the disutility of working, Ω_a is the utility weight of bequests, and $a_{j+1}^b \equiv (1 + r(1 - \tau^k))a_{j+1}$ are next-period bequests. Equation (13) belongs to the class of balanced growth preferences characterised by Boppart and Krusell (2020) that allows for falling hours worked in the long run; the income effect of higher wages on leisure dominates the substitution effect

if $\sigma > 1$, leading to a decline in hours worked when long-run wage growth is positive. These preferences are summed into an expected lifetime utility function of the form

$$\mathbb{E} \left[\sum_{j=\iota}^J \beta^{j-\iota} \left(\prod_{k=\iota}^{j-1} s_k \right) u(c_j, h_j, a_{j+1}^b) \right], \quad (14)$$

where β is the discount factor and expectations are taken over idiosyncratic productivity.

Households maximise expected lifetime utility (14) by choosing consumption, hours worked, and their age of retirement. At the start of each period, households observe their state $x'_j = (a_j, \eta, R_{j-1})$ consisting of current wealth and idiosyncratic productivity and the previous-period retirement status, and subsequently make their retirement decisions. Consumption and hours worked are then chosen in a second stage conditional on the post-retirement decision state $x_j = (a_j, \eta, R_j)$. Retirement is an absorbing state in which labour supply is zero and the household problem reduces to a standard consumption-savings choice.

Formally, denote the retirement choice by a discrete variable d equal to 1 if choosing to remain in the labour force and 0 otherwise and let the retirement status evolve by $R_j = R_{j-1} + d_j$, starting with $R_{\iota-1} = \iota$. This convention implies that R_j equals the household's retirement age once retirement occurs, and that $R_j = j + 1$ while the household continues to work (the earliest age at which it could subsequently retire). An optimal retirement policy is then a function $d_j(x'_j)$ that solves the first-stage problem

$$V_j(x'_j) = \max_{d_j \in D(R_{j-1})} \{v_j(x_j)\} \quad (15)$$

subject to the motion of the retirement status and to

$$D(R_{j-1}) = \begin{cases} \{0, 1\} & \text{if } R_{j-1} = j, \\ \{0\} & \text{if } R_{j-1} < j, \end{cases}$$

where $V_j(x'_j)$ and $v_j(x_j)$ are the pre and post retirement decision value functions at age j . Optimal policies for consumption, savings, and hours worked are functions $c_j(x_j)$, $a_j(x_j)$, and $h_j(x_j)$ that solve the second-stage problem

$$v_j(x_j) = \max_{c_j, h_j} \left\{ u(c_j, h_j, a_{j+1}^b) + s_j \beta \mathbb{E}[V_{j+1}(x'_{j+1}) \mid \eta] \right\} \quad (16)$$

subject to the budget constraint (9), the time constraints $h_j \in [0, 1]$ if working and $h_j = 0$ if retired, and the borrowing constraint $a_{j+1} \geq 0$.

3.3 Production

The competitive final-good sector hires labour L_t at the wage w_t and buys a continuum of intermediate capital inputs $\{k_{it}\}$ indexed by i at prices $\{p_{it}\}$ to produce output Y_t . Production follows the Cobb-Douglas function

$$Y_t = L_t^{1-\alpha} \left(\int_0^{z_t} k_{it}^\rho di \right)^{\frac{\alpha}{\rho}}, \quad 0 < \alpha, \rho < 1, \quad (17)$$

where z_t is a measure of the intermediate varieties available at time t . Intermediate-good firms have a linear production function that converts capital into intermediate inputs one for one. Capital is rented

from households at the rate $r_t + \delta_k$, where δ_k is the capital depreciation rate. Each firm in the intermediate sector has a patent for their own variety and acts as a monopolist. Conditional on having a patent, a firm j consequently maximises operating profits $\pi_{jt} = (p_{jt} - r_t - \delta_k)k_{jt}$ subject to the final-good producer's demand for j . By profit maximisation in the final-good sector, this demand constraint is given by

$$p_{jt} = \alpha \frac{Y_t}{k_{jt}} \frac{k_{jt}^\rho}{\int_0^{z_t} k_{it}^\rho di}. \quad (18)$$

By symmetry, all firms charge the same mark-up over marginal cost, $p_{it} = p_t = \frac{1}{\rho} (r_t + \delta_k)$ and therefore sell the same quantity k_t and earn the same profits $\pi_t = (1 - \rho)p_t k_t$.

3.4 The R&D Sector

The R&D sector exhibits free entry and a unit measure of innovators invests an amount Q_t of final output to develop new designs of intermediate goods.¹¹ Patents for new designs are sold to prospective intermediate-good firms at the end of a period t for a one-off price $P_{z,t+1}$, which then convert the designs into usable input in period $t + 1$. These patent purchases are financed by households in exchange for equity ownership in the intermediate-good firms. There is free entry into R&D and, like the simple model in [Section 2](#), innovation is characterised by an overall production function as in Jones (1995):

$$F(Q_t) = \bar{v}_t Q_t = \nu Q_t^\lambda z_t^\phi, \quad 0 < \lambda \leq 1, \phi < 1. \quad (19)$$

The productivity factor $\bar{v}_t \equiv \nu Q_t^{\lambda-1} z_t^\phi$ incorporates duplication effects (via λ) and knowledge spillovers (via ϕ) that impact the aggregate development but are external to individual innovators. R&D firms thus choose expenditures Q_t to maximise profits $(P_{z,t+1} \bar{v}_t - 1)Q_t$, which together with free entry yields the zero-profit condition

$$Q_t = P_{z,t+1} \nu Q_t^\lambda z_t^\phi. \quad (20)$$

As in Comin and Gertler (2006) and similar papers, an intermediate-good firm is not infinitely lived; each period, a fraction δ_z of firms become obsolete.¹² The law of motion of new intermediates is therefore $z_{t+1} = (1 - \delta_z)z_t + F(Q_t)$, thus implying the gross growth rate of intermediate varieties

$$1 + g_{zt} = 1 - \delta_z + \nu Q_t^\lambda z_t^{\phi-1}. \quad (21)$$

Lastly, a prospective intermediate producer enters only if it is profitable to do so. That is, the firm enters if the sum of expected discounted flow profits π exceeds the fixed cost P_z of purchasing a patent. Free entry into the intermediate-good sector drives the profitability of entry to zero. This is equivalent to saying that the following no-arbitrage condition holds in equilibrium:

$$r_t = \frac{\pi_t + \Delta P_{zt} - \delta_z P_{z,t+1}}{P_{zt}}, \quad (22)$$

¹¹ Although innovators are not modelled explicitly, this framework allows innovative capacity to vary with age because R&D input—final output—is itself produced using workers with an age-specific productivity profile $\{\varepsilon_j\}_{j=1}^J$. Shifts in the workforce age composition therefore feed into R&D productivity through this channel. [Section 7](#) makes this mechanism more direct by considering a specification in which labour is used as the direct R&D input.

¹² This feature is primarily technical to calibrate the level of R&D investment. We can also set $\delta_z < 0$ to generate exogenous growth, as in the standard neoclassical model, in addition to that created through R&D.

where $\Delta P_{zt} \equiv P_{z,t+1} - P_{zt}$ is the change in the patent price in period t .

3.5 The Public Sector

The public sector engages in three activities: it redistributes assets from deceased individuals; it collects taxes on consumption, capital gains, and wages via the tax rates τ_t^c , τ_t^k , and τ_t^w to finance public consumption G_t ; and it operates a pay-as-you-go social security system, which is financed by contribution rates τ_t^b on labour earnings. The budget constraint of each public sector activity is independent of the other two and always balances. In the baseline model, budget balance is achieved through endogenous adjustments in the bequest transfers tr , public consumption G_t , and social security contribution rates τ_t^b . In this setup, an ageing population requires higher contribution rates to sustain pension benefits. Compared to the alternative of constant tax rates and benefit cuts, this is the most growth-pessimistic arrangement, since it both raises taxation of working households (reducing the ability to save) and maintains more generous replacement rates (reducing the incentive to save and to delay retirement).

Pensions are independent of earnings history but depend on the age of retirement.¹³ Specifically, a household with retirement status R receives a pension that is scaled by a factor $ps(R)$ relative to a base-level benefit received if retiring at some normal retirement age R^{norm} , thus allowing for early retirement penalties and delayed retirement credits. The base-level benefit is a fraction μ of average gross labour income $w_t \bar{\ell}_t$, where $\bar{\ell}_t$ is the average number of efficiency units per worker. An age- j household therefore receives a pension transfer

$$b_t(R_j) = \begin{cases} ps(R_j) \mu w_t \bar{\ell}_t & \text{if } j \geq \max\{R^{min}, R_j\}, \\ 0 & \text{if } j < \max\{R^{min}, R_j\}, \end{cases} \quad (23)$$

where no pension is paid out to retirees younger than some threshold age R^{min} or to workers.

3.6 Equilibrium

In any period t , the state of the economy is pinned down by the aggregate capital stock $K_t \equiv \int_0^{z_t} k_{it} di$, the number of intermediate varieties z_t , the patent price P_{zt} , and measures $\{\Phi_{jt}(x)\}_{j=t}^J$ characterising the distribution of households after their retirement decisions are made. Since the state of a household is determined by their asset wealth, idiosyncratic productivity, and retirement status, the state space for the household measures is given by $X = \mathbb{R}_+ \times H \times \{1, \dots, J+1\}$, where H denotes the state space of the Markov chain for idiosyncratic productivity. Given a demographic evolution and initial conditions $K_0, z_0, P_{z,0}, \{\Phi_{j0}(x)\}_{j=1}^J$, we then have:

Definition 1 (General equilibrium). An equilibrium consists of paths for household decision rules $\{d_{jt}(\cdot), c_{jt}(\cdot), a_{jt}(\cdot), h_{jt}(\cdot)\}_{t=0}^\infty$ and measures $\{\Phi_{jt}(\cdot)\}_{t=0}^\infty$ for all ages j , prices and profits $\{r_t, w_t, \pi_t, P_{zt}\}_{t=0}^\infty$, taxes and transfers $\{\tau_t^c, \tau_t^k, \tau_t^b, \tau_t^w(\cdot), tr, b_t(\cdot)\}_{t=0}^\infty$, and aggregate quantities such that for all periods t :

- (i) Household decision rules solve problems (15) and (16).
- (ii) Profit maximisation of final and intermediate good firms yields the consolidated production function

$$Y_t = K_t^\alpha (Z_t L_t)^{1-\alpha}, \quad \text{where} \quad Z_t \equiv z_t^{\frac{\alpha}{1-\alpha} \frac{1-\rho}{\rho}}, \quad (24)$$

¹³ Earnings-dependent pension introduces another continuous state variable in the household problem and I abstract from this feature to avoid the added computational complexity that it entails, as is common in Auerbach and Kotlikoff (1987) models.

with corresponding factor prices and profits

$$w_t = (1 - \alpha) \frac{Y_t}{L_t}, \quad r_t = \alpha \rho \frac{Y_t}{K_t} - \delta_k, \quad \pi_t = \alpha(1 - \rho) \frac{Y_t}{z_t}. \quad (25)$$

- (iii) The measure of intermediate varieties z_t , the patent price P_{zt} , and aggregate R&D investment Q_t satisfy [Equations \(20\) to \(22\)](#).
- (iv) The public sector balances its budgets:

$$G_t = \tau_t^c C_t + \tau_t^k r_t A_t + \sum_{j=\iota}^J N_{jt} \int_X \tau^w(w_t \ell_{jt}(x)) w_t \ell_{jt}(x) d\Phi_{jt}, \quad (26)$$

$$\tau_t^b w_t L_t = \sum_{j=\iota}^J N_{jt} \int_X b_t(R) d\Phi_{jt}, \quad (27)$$

$$tr_{t+1} = \frac{1 + r_{t+1}(1 - \tau_{t+1}^k)}{\sum_{j=\iota}^J N_{j,t+1}} \sum_{j=\iota}^J (1 - s_{jt}) N_{jt} \int_X a_{jt}(x) d\Phi_{jt}, \quad (28)$$

where $C_t = \sum_{j=\iota}^J N_{jt} \int_X c_{jt}(x) d\Phi_{jt}$ is aggregate consumption and $A_{t+1} = \sum_{j=\iota}^J (1 + m_{jt}) N_{jt} \int_X a_{jt}(x) d\Phi_{jt}$ is aggregate wealth.

- (v) Labour, capital, and goods markets clear:

$$L_t = \sum_{j=\iota}^J N_{jt} \int_X \ell_{jt}(x) d\Phi_{jt}, \quad (29)$$

$$A_t = K_t + P_{zt} z_t, \quad (30)$$

$$Y_t + A_{t+1}^M = C_t + G_t + [K_{t+1} - (1 - \delta_k) K_t] + Q_t, \quad (31)$$

where $A_{t+1}^M = \sum_{j=\iota}^J m_{jt} N_{jt} \int_X a_{jt}(x) d\Phi_{jt}$ is the net wealth brought by migrants.

- (vi) For a given Markov kernel Π and for all Borel sets $S = \mathcal{A} \times \mathcal{H} \times \mathcal{R}$ on X (and dropping time subscripts), the household distributions evolve according to

$$\Phi_{j+1}(S) = \int_{X_W} \left[\int_{\mathcal{H}} d_{j+1}(x') \Pi(\eta, d\eta') \right] d\Phi_j + \int_{X_R} \left[\int_{\mathcal{H}} (1 - d_{j+1}(x')) \Pi(\eta, d\eta') \right] d\Phi_j, \quad (32)$$

where $X_W = \{x \in X : a_j(x) \in \mathcal{A}, R + 1 \in \mathcal{R}\}$ and $X_R = \{x \in X : a_j(x) \in \mathcal{A}, R \in \mathcal{R}\}$, and where $x' = (a_j(x), \eta', R)$, $\eta' \in \mathcal{H}$, are the relevant pre-retirement decision states at age $j + 1$. If Γ denotes the unique invariant distribution of Π , the initial household distribution is equal to

$$\Phi_{\iota}(\{0\} \times \mathcal{H} \times \{R\}) = \begin{cases} \int_{\mathcal{H}} d_{\iota}(0, \eta, \iota) d\Gamma & \text{if } R = \iota + 1, \\ \int_{\mathcal{H}} (1 - d_{\iota}(0, \eta, \iota)) d\Gamma & \text{if } R = \iota, \end{cases} \quad (33)$$

and zero for all other asset levels and retirement statuses. ◀

As in Romer (1990), the model features endogenous growth in total factor productivity Z_t through changes in the measure of intermediate varieties z_t . As in Jones (1995), this growth is semi-endogenous in that it is exogenously determined by the rate of population growth along a balanced growth path.

Definition 2 (Stationary equilibrium). A stationary equilibrium, or steady state, is an equilibrium in which all variables grow at constant (possibly zero) rates and all growth rates are determined by the population growth rate $1 + n \equiv \frac{N_{t+1}}{N_t}$. In particular, the growth rate of new intermediate-good firms is

$$1 + g_z = (1 + n)^\chi, \quad \text{where} \quad \chi \equiv \frac{\lambda}{1 - \phi - \lambda \frac{1+\theta}{1+\theta\sigma} \frac{\alpha}{1-\alpha} \frac{1-\rho}{\rho}} > 0. \quad (34)$$

The growth rates of TFP, output per capita, and hours per worker are similarly

$$1 + g_z = (1 + n)^{\gamma_z}, \quad 1 + g_y = (1 + n)^{\gamma_y}, \quad 1 + g_h = (1 + n)^{\gamma_h},$$

where $\gamma_z \equiv \chi \frac{\alpha}{1-\alpha} \frac{1-\rho}{\rho}$, $\gamma_y \equiv \frac{1+\theta}{1+\theta\sigma} \gamma_z$, and $\gamma_h \equiv \frac{\theta(1-\sigma)}{1+\theta\sigma} \gamma_z = \gamma_y - \gamma_z$. ◀

Online Appendix A derives the growth rates in Definition 2. Note that TFP growth reduces to Equation (2) in the simple model if we consider a steady state with constant hours worked (via log preferences, $\sigma \rightarrow 1$) and a substitution parameter for intermediate goods equal to the capital share parameter ($\rho = \alpha$).

4 Numerical Experiment and Computation

As a quantitative exercise, I consider a transition from 1950 to 2100 where demographic change is the only exogenous driver. The transition mimics observed and projected population trends and eventually converges to a steady state consistent with the projected fertility, mortality, and migration rates by the end of the twenty-first century. To reduce the impact of the initial steady-state assumption on the period of interest, I initialise the model in a stationary equilibrium in 1900. In 1901, the economy is shocked by the demographic transition, after which agents have perfect foresight over aggregate variables until the new steady state. The 1901–1950 period serves as a calibration phase, during which model moments are matched to data to pin down key parameters.

To isolate the transitional effects of demographic change, I impose a common long-run population growth rate in both the initial and terminal steady states. Since long-run per-capita growth is pinned down by population growth by Definition 2, the equilibrium path would otherwise conflate transitional effects with a mechanical steady-state growth shift, obscuring the contribution of the demographic transition itself. As detailed in Section 5.1, the demographic path between the steady states remains consistent with the data and replicates observed population dynamics (see Figure 2). The normalization only removes steady-state growth differences, so deviations from trend during the transition can be interpreted cleanly.

Numerically, I solve the household problem using the discrete-continuous endogenous grid method of Iskakov *et al.* (2017) and approximate the household distributions with histograms over wealth, idiosyncratic productivity, and retirement status. Iskakov *et al.* extend Carroll's (2006) endogenous grid method to incorporate discrete choices, which substantially reduces the computational burden associated with the retirement decision. The overall model equilibrium is obtained via an Anderson acceleration

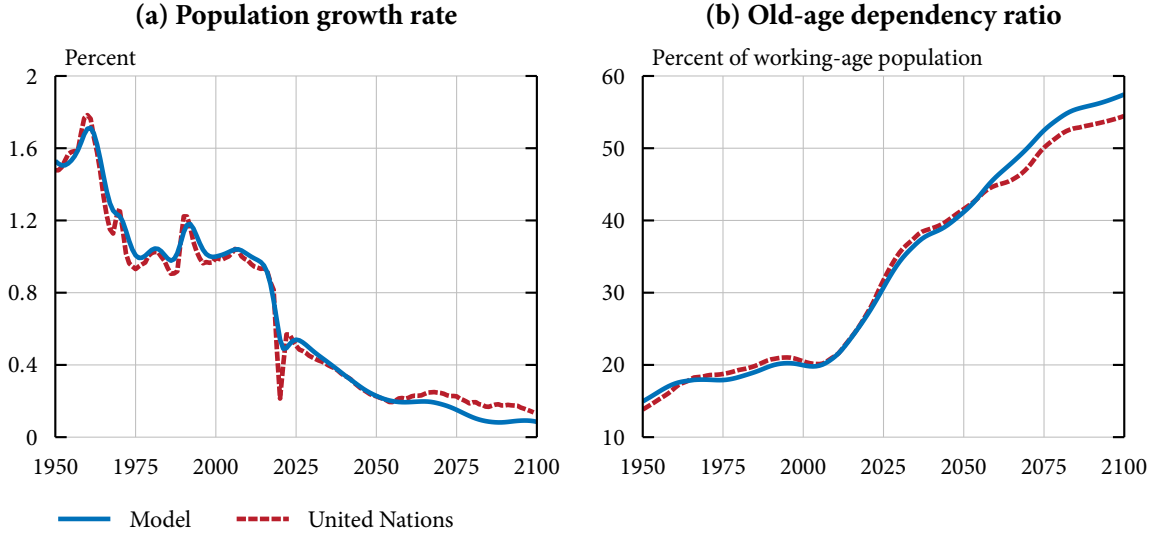


Figure 2. Demographic evolution.

procedure that iterates over paths of interest rates (r_t), pension tax rates (τ_t^b), average labour supplies ($\bar{\ell}_t$), bequests (tr_t), and growth rates of intermediate varieties (g_{zt}) until all equilibrium conditions hold.

5 Bringing the Model to the Data

The model is calibrated to be consistent with key features of the US economy. I first set a subset of model parameters externally based on exogenous estimations and sources. With these in hand, I then estimate the remaining parameters internally by matching transition path moments of the calibration phase (pre-1950) with US macro data. The results are summarised in Table 3 and discussed in detail below.

5.1 Demographics

The demographic path is constructed from annual age-specific survival, fertility, and migration data. For 1950–2100, I use the United Nations (2024) World Population Prospects. For earlier years, I collect survival rates from Bell and Miller (2005), fertility rates from Heuser (1976) and Gapminder Foundation (2024), and use US Department of Homeland Security (2020) immigration data as a proxy for aggregate net migration. The age distribution of migration is assumed throughout to match the US Census Bureau’s (2023) population projection. All series are converted into rates per person and age group using Census Bureau intercensal population tables and the UN World Population Prospects where applicable, and then smoothed with an HP filter.

To implement the constant long-run growth normalization from Section 4, I adjust the steady-state migration rates by a common term in each steady state so that the initial and final steady states exhibit identical population growth.¹⁴ The 2100 demographic variables are already consistent with near-zero steady-state growth, so I set this common population growth rate to exactly zero. This normalization has no bearing on the quantitative analysis: the adjustments only modify migration rates at the steady-state boundaries; the survival and fertility rates remain fully consistent with the underlying data and the 1950–2100 transition

¹⁴ If x is a common migration rate shifter, a steady-state population satisfies $N_{j+1} = \frac{s_j + m_j + x}{1+n} N_j = \left(\prod_{k=0}^j \frac{s_k + m_k + x}{1+n} \right) N_0$. Consistency with fertility rates requires that $(1+n)N_0 = \sum_{j=0}^J f_j N_j$. Combining these yields the condition $1+n = \sum_{j=0}^J \left(\prod_{k=0}^{j-1} \frac{s_k + m_k + x}{1+n} \right) f_j$, which can be solved for x numerically for a given population growth rate $1+n$.

Table 3. Calibrated parameters of the baseline model.

Parameter	Description	Value	Source or target
<i>Households</i>			
ι	Initial adult age	20	Children between 0–19
J	Maximum age	99	Certain death at 100
β	Discount factor	1.011	Capital-output ratio
ψ	Leisure weight	80.23	Hours per person aged 20–64
Ω_A	Bequest weight	16.55	Old-age asset wealth
σ	Inverse IES	1.75	Boppart and Krusell (2020)
θ	Frisch elasticity	0.5	Chetty <i>et al.</i> (2011)
ζ	Household consumption scale	1.49	Browning and Ejrnæs (2009)
$\{\omega_i\}_{i=0}^{\iota-1}$	Utility weights on dependants	Fig. F.4 ^a	Browning and Ejrnæs (2009)
<i>Individual productivity</i>			
$\{\varepsilon_j\}_{j=\iota}^J$	Deterministic productivity	Fig. F.5 ^a	PSID
φ	Persistence shock	0.97	Heathcote <i>et al.</i> (2010)
σ_ε^2	Variance shock	0.02	Heathcote <i>et al.</i> (2010)
<i>Production</i>			
α	Intermediate goods share	0.35	Labour share of income
ρ	Elasticity of substitution	0.67	Profit share of income
δ_k	Capital depreciation rate	0.034	Depreciation/output
<i>R&D</i>			
δ_z	Firm obsolescence rate	0.002	IPP depreciation/output
ν	R&D productivity	0.009	$z = Z = 1$ in initial period
λ	Duplication externality	0.75	Comin and Gertler (2006)
ϕ	Knowledge spillovers	−0.068	$g_Q = 5.97\% \implies g_Z = 1.11\%$
<i>Social security</i>			
μ	Replacement rate	0.413	Clingman <i>et al.</i> (2021)
R^{norm}	Normal retirement age	65	Social Security Administration
R^{min}	Lowest retirement age	62	Social Security Administration
$ps(R_j)$	Early/delayed scaling	0.8–1.15	Social Security Administration
<i>Taxes</i>			
τ^c	Consumption tax rate	0.080	BEA national accounts
τ^k	Capital gains tax rate	0.383	BEA national accounts
$\tau^w(w\bar{\ell})$	Income tax rate at mean income	0.116	BEA national accounts
κ_0	Asymptotic income tax rate	0.631	OECD tax database
κ_1	Income tax progressivity	0.574	OECD tax database
κ_2	Income tax scale parameter	0.505	OECD tax database
κ_3	Income tax rate at zero income	−0.205	OECD tax database

^a Available in [Online Appendix F](#).

path is data-driven throughout. [Figure 2](#) plots the implied demographic development and confirms that the calibration closely matches United Nations estimates during the period of interest.

5.2 Households

Households start their economic lives upon turning 20 and die with certainty at 100. I set the Frisch elasticity of labour supply θ to 0.5, as recommended by Chetty *et al.* (2011) along the intensive margin. This

value is also consistent with Domeij and Flodén (2006a), who explicitly account for the bias arising from uninsurable income risk and borrowing constraints. The inverse intertemporal elasticity of substitution, σ , is calibrated following Boppart and Krusell (2020), who present macro evidence that 2 percent productivity growth in the long run implies a fall in hours worked by roughly 0.4 percent per year. Since the steady-state growth rates of hours worked and technology in Definition 2 are linked via $1 + g_h = (1 + g_Z)^{\frac{\theta(1-\sigma)}{1+\theta\sigma}}$, this empirical pattern suggests that $\theta(1 - \sigma)/(1 + \theta\sigma) \approx -0.2$. Given $\theta = 0.5$, this generates a σ of 1.75. While based on long-run macro trends, these parameter values are also consistent with micro evidence: Heathcote, Storesletten and Violante (2014) consider an incomplete-markets model with similar preferences and estimate $(\sigma, \theta) = (1.71, 0.46)$ using US micro data.

The adult equivalence weights $\omega_i^{\frac{\zeta}{\zeta-(1-\sigma)}}$ and the household scale parameter ζ are based on Browning and Ejrnæs (2009), who estimate equivalence scales for low- and high-educated households that include age-specific weights for children and economies of scale. Browning and Ejrnæs use UK data, though Fernández-Villaverde and Krueger (2007) demonstrate with US data that a similar age-varying equivalence scale explains the hump shape of life-cycle consumption better than a traditional scale. The equivalence weights are set to the average of Browning and Ejrnæs’ estimates for the two education groups. Their economies-of-scale parameter, call it ζ_{BE} , is linked to the model here via $\zeta_{BE} = \frac{\zeta-(1-\sigma)}{\zeta\sigma}$. Given $\sigma = 1.75$ and choosing $\zeta_{BE} = 0.86$, again the average of Browning and Ejrnæs’ estimates, then yields $\zeta = 1.49$.

I estimate the deterministic age-efficiency profile $\{\varepsilon_j\}_{j=t}^J$ by a fixed-effects regression of real log wages on a quadratic in age using 1968–2019 Panel Study of Income Dynamics data (see Online Appendix B for details). The resulting profile features a standard hump shape that peaks between the ages of 40 and 50. The idiosyncratic productivity shocks are assumed to follow the AR(1) process $\log \eta' = \varphi \log \eta + \varepsilon$, where $\varepsilon \sim N(0, \sigma_\varepsilon^2)$. The persistence parameter φ and error variance σ_ε^2 are set to 0.97 and 0.02, following the evidence in Heathcote, Storesletten and Violante (2010). This process is discretised into a five-state Markov chain using Rouwenhorst’s method (Kopecky and Suen, 2010).

5.3 R&D Production Function

The R&D production function (19) contains three parameters: the productivity level ν , the duplication externality λ , and the knowledge spillover ϕ . The first is a scale parameter that I set so that $z = Z = 1$ in the initial steady state. The duplication externality λ governs how aggregate R&D effort translates into new ideas: $\lambda = 1$ implies that each unit of input contributes independently, while $\lambda < 1$ captures “standing on toes” effects whereby parallel research efforts overlap. There is no consensus on its empirical value (Bloom *et al.*, 2020), but calibrations by Jones and Williams (2000) and Comin and Gertler (2006) identify a plausible range of 0.5 to 1. I set $\lambda = 0.75$, the midpoint of this range, as a benchmark. The R&D block’s quantitative discipline instead comes from the calibration of ϕ , which I pin down using productivity and investment data below.¹⁵

Following Jones and Williams (2000) and Bloom *et al.* (2020), ϕ is calibrated so that the R&D production function (19) is consistent with the empirical long-run relationship between TFP (Z in the model) and real gross investment in intellectual property products (IPP, Q in the model).¹⁶ Figure 3 shows that US

¹⁵ Section 7 also varies ϕ to span the full range of long-run growth exponents γ_Z and γ_Y identified by Jones (2002), which effectively limits the influence of this λ choice.

¹⁶ Besides formal R&D, IPP investment includes nonrival goods such as computer software and entertainment, literary, and artistic originals. This broader measure is arguably closer to the model’s notion of innovation, so I use total IPP investment rather than formal R&D as the relevant calibration variable.

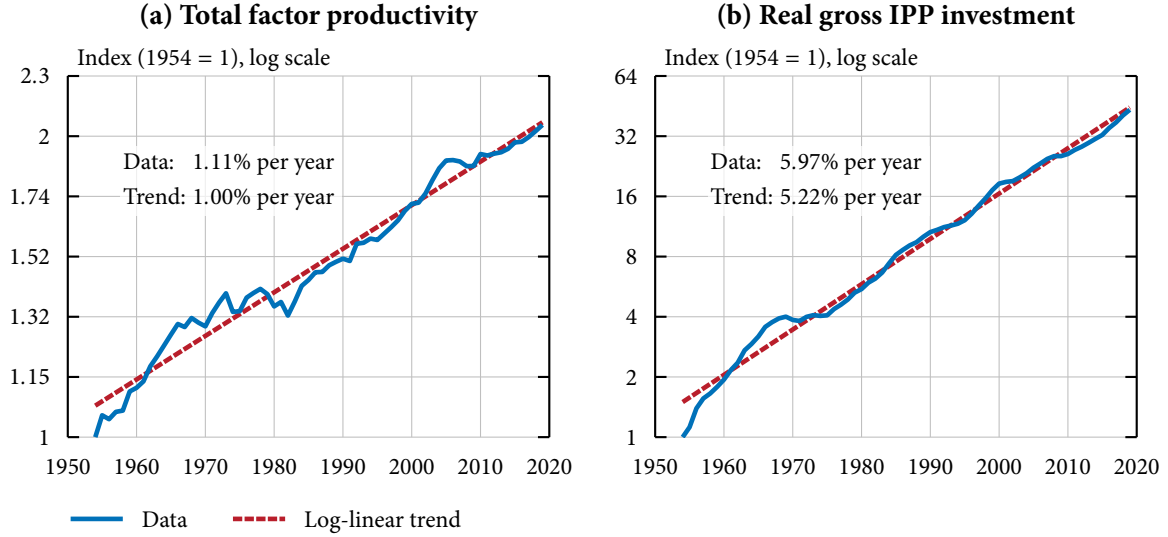


Figure 3. US TFP and aggregate IPP investment in the data.

Source. TFP from the Penn World Table (Feenstra, Inklaar and Timmer, 2015), expressed as Harrod-neutral technological change using labour shares from the same source. IPP investment from BEA NIPA Table 5.2.3.

TFP and IPP investment exhibit roughly constant growth rates of 1.11 and 5.97 percent per year since the 1950s. Since TFP and the measure of intermediate varieties are linked in the model via $Z = z^{\frac{\alpha}{1-\alpha} \frac{1-\rho}{\rho}}$, the R&D production function can replicate such constant-growth paths provided that $\nu Q^\lambda z^{\phi-1}$ remains fixed when TFP Z and investment Q grow at constant rates, which in turn requires $\lambda g_Q = (1 - \phi)g_Z$. Combining this restriction with $g_Z = \frac{\alpha}{1-\alpha} \frac{1-\rho}{\rho} g_Z$ gives a mapping between the data and the spillover parameter:

$$\phi = 1 - \lambda \frac{\alpha}{1-\alpha} \frac{1-\rho}{\rho} \frac{g_Q}{g_Z}.$$

Plugging in the calibrated values for λ , α , and ρ together with the empirical growth rates in Figure 3 yields $\phi = -0.068$. The negative value implies that research productivity declines as the knowledge frontier expands, consistent with the evidence of Bloom *et al.* (2020) that ideas are getting harder to find.¹⁷ This calibration also generates steady-state growth exponents γ_Z and γ_Y of 0.22 and 0.18, close to the mid-range of Jones's (2002) estimates.

5.4 Public Sector

I set the gross pension replacement rate μ to 0.413 based on Clingman, Burkhalter and Chaplain's (2021) estimates for average-income workers retiring at the normal retirement age. The normal retirement age is $R^{norm} = 65$ and the earliest pension eligibility age is $R^{min} = 62$. The pension adjustment schedule $ps(R)$ mirrors the US Social Security system: benefits are reduced by 6 2/3 percent per year for the first three years of early retirement and by 5 percent for each additional year, while delayed retirement raises benefits by 3 percent per year up to age 70. No further adjustment applies beyond age 70.¹⁸

¹⁷ This particular value of ϕ is nevertheless larger than what Bloom *et al.* (2020) find for the aggregate US economy. This is because they estimate ϕ under the implicit assumption that $\rho = \alpha$. Imposing the same restriction here yields a similar value.

¹⁸ These are the exact rules used by the Social Security Administration for cohorts born before 1924. Later cohorts have higher normal retirement ages and more generous delayed retirement credits, though Online Appendix E shows that the quantitative results are insensitive to a more accurate development of R^{norm} and $ps(R)$.

The income tax rate $\tau^w(e)$ at household earnings e is parametrised by the Gouveia and Strauss (1994) form:

$$\tau^w(e) = \kappa_0 \left[1 - \left(\kappa_2 \left(\frac{e}{\bar{e}} \right)^{\kappa_1} + 1 \right)^{-\frac{1}{\kappa_1}} \right] + \kappa_3, \quad (35)$$

where κ_0 controls the asymptotic tax rate $\lim_{e \rightarrow \infty} \tau^w(e) = \kappa_0 + \kappa_3$ and κ_1 and κ_2 determine the degree of tax progressivity. Equation (35) augments Gouveia and Strauss' original function with average labour earnings $\bar{e}$, which makes the tax function invariant to units of measurement, and the shifter κ_3 , which allows for non-zero marginal tax rates at zero earnings.

To calibrate taxes, I first construct tax rates on consumption, capital, and labour income using national accounts data from the BEA by dividing aggregate tax revenues by their corresponding tax base. Averaging these rates between 1950 and 2022 yields τ^c , τ^k , and $\tau^w(\bar{e})$, the latter denoting the income tax rate at average earnings. I then estimate $\{\kappa_0, \kappa_1, \kappa_2, \kappa_3\}$ using the OECD tax database, which reports income-specific effective tax rates incorporating federal, state, and local taxes as well as major deductions and credits. These estimates are available annually from 2000 onward, but only for a small set of representative income levels. I therefore replicate the OECD methodology to construct tax rates over a full income grid for each year and fit (35) to the resulting schedule. Finally, to ensure consistency with the aggregate data, I shift the estimated tax curve so that the implied rate at average earnings, $\tau^w(\bar{e})$, matches the national accounts estimate. Online Appendix C provides further details.

5.5 Internally Calibrated Parameters

Seven parameters are estimated using the simulated method of moments procedure for Auerbach and Kotlikoff (1987) models proposed by Ludwig (2005): the discount factor (β), the utility weights on labour supply and bequests (ψ , Ω_A), the depreciation rates for capital and R&D (δ_k , δ_z), and the production function parameters (α , ρ). The calibration matches model-generated moments to their empirical counterparts over the pre-1950 calibration phase of the transition path. The pre-1950 focus here is deliberate: because the analysis begins in 1950, post-1950 moments are predictions of the model, so using them as calibration targets would amount to fitting parameters to the very dynamics under study. This estimation is a computationally costly procedure, since each evaluation of the moment criterion requires recomputing the full transition path and thus forces calibration and simulation to be performed jointly.

While estimated jointly, each parameter corresponds to a moment that is most informative for its identification. In the order above, these moments and their sample years are: the capital-output ratio (1901–1949), hours worked per person aged 20–64 (1901–1949), the average net worth of individuals aged 65+ relative to those aged 55–64 (1950), consumption of non-IPP capital relative to GDP (1901–1949), consumption of IPP capital relative to GDP (1929–1949), and the labour and profit shares of national income (1901–1949). Because official US data are limited for the first half of this period, I construct the empirical moments by splicing historical sources with BEA and Bureau of Labor Statistics data. One exception is IPP depreciation, which, to my knowledge, is not available before the BEA series, thus restricting this moment to 1929–1949. Online Appendix D provides further details and shows that the model successfully matches the targeted low-frequency moments over the pre-1950 period.

5.6 Untargeted Moments

Before turning to the main results, I briefly examine how the calibrated model performs along margins that were not targeted in estimation, both at the onset in 1950 and along the subsequent transition. The

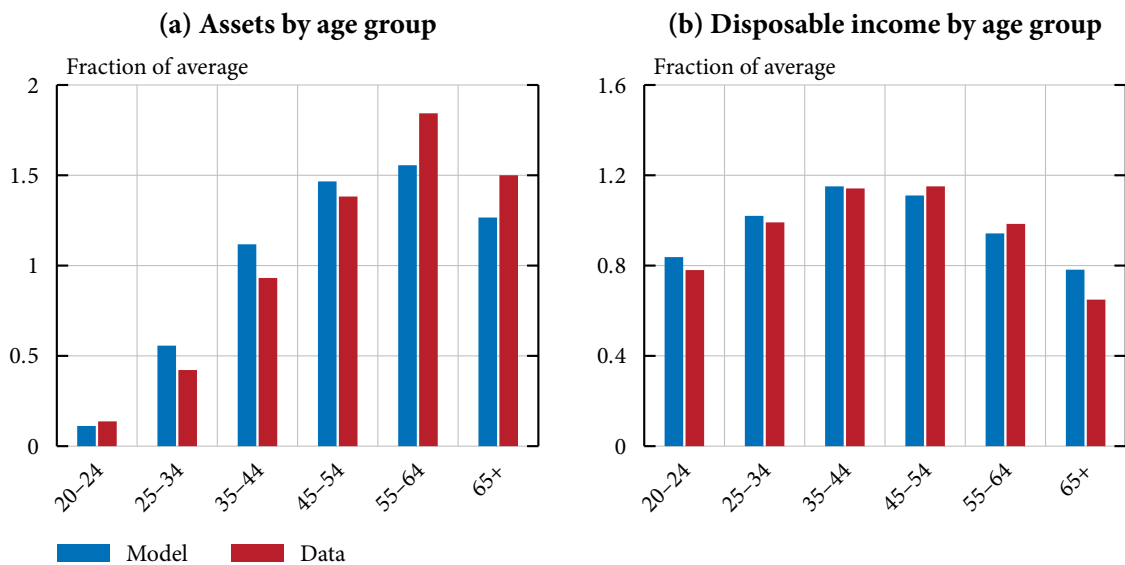


Figure 4. Life-cycle profiles in 1950 in the model and the data.

Source. Net worth from the 1949 Survey of Consumer Finances (Goldsmith, 1955, Table XLIII). Disposable income from the 1950 Survey of Consumer Expenditures (US Bureau of Labor Statistics, 1957, Table 1-1).

post-1950 comparisons here are best read as sanity checks rather than formal validation exercises: post-1950 outcomes are results of the numerical experiment, not calibration targets, so evaluating the model on how well they match the data would be circular. The purpose of the paper is to isolate the contribution of demographic change to post-1950 dynamics, and the difficulty of identifying that contribution directly from the data is precisely what motivates the structural approach in the first place. The aim here is therefore the more modest one of showing that a parametrisation disciplined entirely by pre-1950 data and external sources delivers a transition path that is internally coherent and externally plausible.¹⁹

Figure 4 compares predicted and empirical cross-sectional age profiles of assets and after-tax income in 1950. The model replicates the overall hump shape of the asset profile well, with the calibrated bequest motive aligning peak and retirement-age holdings with the data. The model somewhat overpredicts asset accumulation early in life and correspondingly underpredicts middle-age accumulation. This pattern primarily reflects the calibrated discount factor, which is disciplined by the aggregate capital-output ratio and broadly consistent with existing micro evidence on intertemporal preferences in the presence of mortality risk, such as Hurd (1989). Given the aggregate focus of the paper, matching the overall saving behaviour of the economy is prioritised over further fine-tuning of the exact shape of the asset profile. The corresponding after-tax income profile, meanwhile, matches closely with the data throughout.

Figure 5a similarly shows that the stylised retirement framework aligns reasonably well with cross-sectional retirement patterns in 1950. At that time, however, Social Security eligibility was limited; as the program expanded, the average retirement age declined—a pattern not reflected in the model. Figure 5b nevertheless reveals that from the 1970s onward, when Social Security approached universal coverage, the model closely tracks the share of retired workers in the data. The discrepancy in earlier years stems precisely from the

¹⁹ The focus on demographic change means that the framework abstracts from several other post-1950 secular trends, such as rising female labour force participation and delayed labour force entry. Such trends undoubtedly influence household behaviour, but matter here only insofar as they interact with demographic change. The comparisons in this section, alongside the robustness of the baseline results to alternative specifications of household preferences, risk, and fiscal policy shown in Online Appendix E, suggest that these interactions are quantitatively limited.

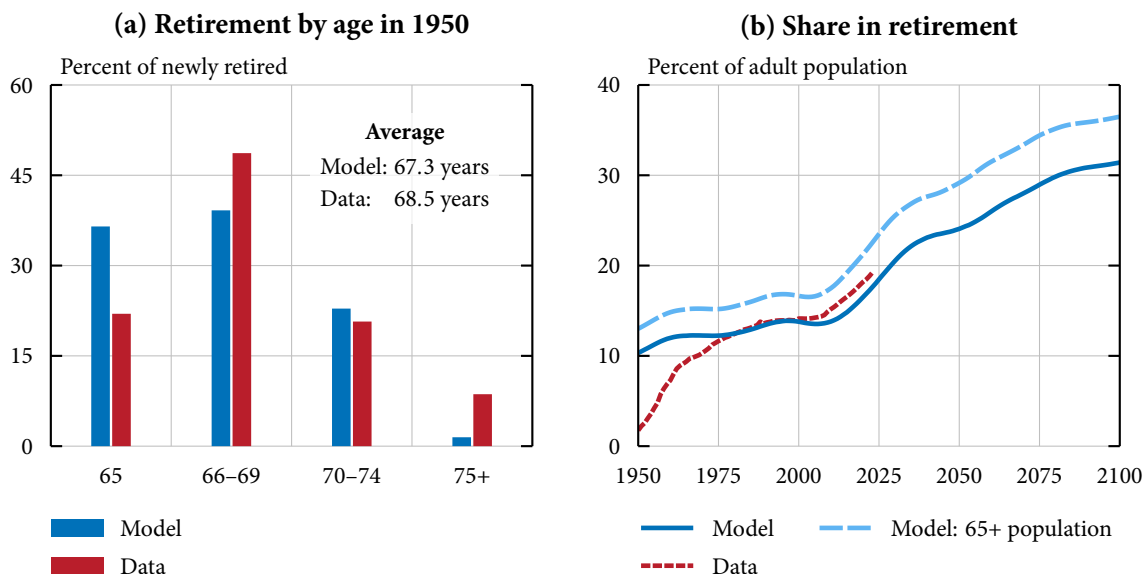


Figure 5. Retirement in the baseline model and the data.

Source. Social Security Administration (2024, Tables 5.B5 and 6.B5).

model’s assumption of universal coverage and time-invariant eligibility rules throughout. Holding these rules fixed removes policy-induced variation from the experiment and isolates the role of demographics more cleanly. The predicted retirement share also exhibits dynamics similar to those of the population share aged 65 and above, suggesting that demographic composition is the primary driver of long-run retirement trends in the model—the same conclusion that the benchmark case without endogenous retirement imposes by assumption.

Most importantly for the paper’s central mechanism, Figure 6 plots a model analogue of Figure 1, comparing 1970–2020 gross saving and investment rates from the model’s US transition path with the OECD panel evidence. The model reproduces the main qualitative pattern in the data: over 1970–2020, a one-year increase in life expectancy at age 20 raises the gross saving rate by 0.57 percentage points and the gross R&D investment rate by 0.15 percentage points, broadly in line with the corresponding OECD slopes of 0.70 and 0.20 despite not being calibration targets. These OECD estimates are of course descriptive and should not be interpreted causally. Even so, this comparison provides a useful first-order check that the sign and magnitude of the savings-R&D channel driving the next section’s results are quantitatively sensible.

6 Quantitative Results

The lack of population growth in the initial and final steady states means that there is no long-run economic growth. The growth impact of the demographic transition is therefore purely transitory. This section illustrates and quantifies this impact and its underlying mechanisms via three quantitative exercises: (i) a growth accounting analysis of per-capita growth in the baseline model, (ii) counterfactual simulations that identify the key demographic factors at work, and (iii) a comparison to an equivalent model without endogenous growth to assess the importance of the TFP channel.²⁰ Throughout, I take “per capita” to mean “per adult equivalent person”, since children only affect the economy through the household size.

²⁰ Additional figures of other variables are available in [Online Appendix F](#).

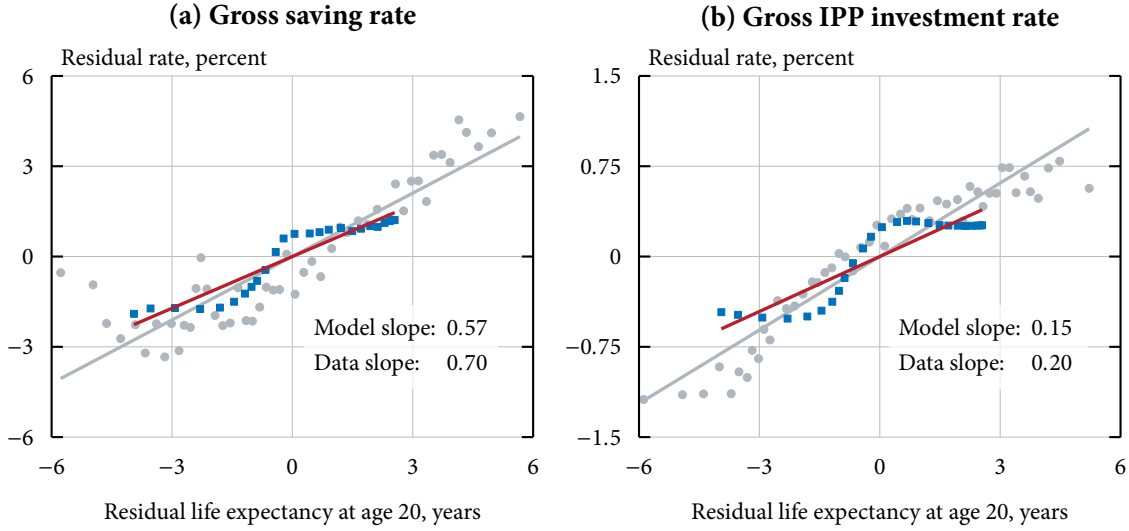


Figure 6. Partial correlations of savings, investment, and life expectancy: model and data.

Notes. The blue squares are predictions from the model between 1970 and 2020, after partialling out population dependency rates (0–14 and 65+ population shares). The red line shows the OLS fit. The gray elements are the OECD data in Figure 1.

The growth accounting exercise decomposes aggregate labour—the total efficiency units supplied by households—into total employment (E), average hours per worker ($\bar{h}$), and average productivity per hour worked ($\bar{\epsilon}$): $L = E \bar{h} \bar{\epsilon}$. Combining this with output per capita, $y = Z \left(\frac{K}{Y}\right)^{\frac{\alpha}{1-\alpha}} \frac{L}{N}$, and log-differencing between subsequent periods yields the growth accounting identity

$$g_y = g_Z + \frac{\alpha}{1-\alpha} g_{K/Y} + g_{E/N} + g_h + g_\epsilon, \quad (36)$$

where g_x denotes the growth rate of a variable x . Equation (36) splits output per capita growth into changes in technology, the capital intensity, the employment rate, average hours per worker, and average productivity per hour worked, which makes it easy to quantify the transitional mechanisms at work.²¹

6.1 Comparative Statics

Following the discussion in Section 2, it is useful to first consider comparative statics of the baseline scenario. Figure 7 displays the overall change in the age structure and the cumulative percentage change in output per capita between the initial and final steady states, which reveals that the change in the population structure raises output per capita by 45 percent. This follows primarily from a substantial increase of 72 percent in TFP relative to its initial level. The capital intensity also increases by 17 percent while the higher share of old-age households in the population reduces the employment rate by 22 percent. The qualitative changes in the underlying channels are therefore precisely as the simple model predicts.

A natural question immediately emerges from Figure 7: why is the magnitude of the change in TFP so much larger than any of the other changes? It is not because an overwhelming majority of household savings are allocated to R&D rather than to capital investments.²² Rather, this reflects the scale effect

²¹ Generally, for any fixed long-run population growth rate $\tilde{n}$, we can identify and decompose transitional growth as in Jones (2002) via $g_y - \gamma_y \tilde{n} = (g_Z - \gamma_Z \tilde{n}) + \frac{\alpha}{1-\alpha} g_{K/Y} + g_{E/N} + (g_h - \gamma_h \tilde{n}) + g_\epsilon$.

²² Capital investment exceeds R&D investment by a factor of 4 during most of the transition and the growth rate of the capital stock relative to TFP is even higher; see Figure F.6 in Online Appendix F.

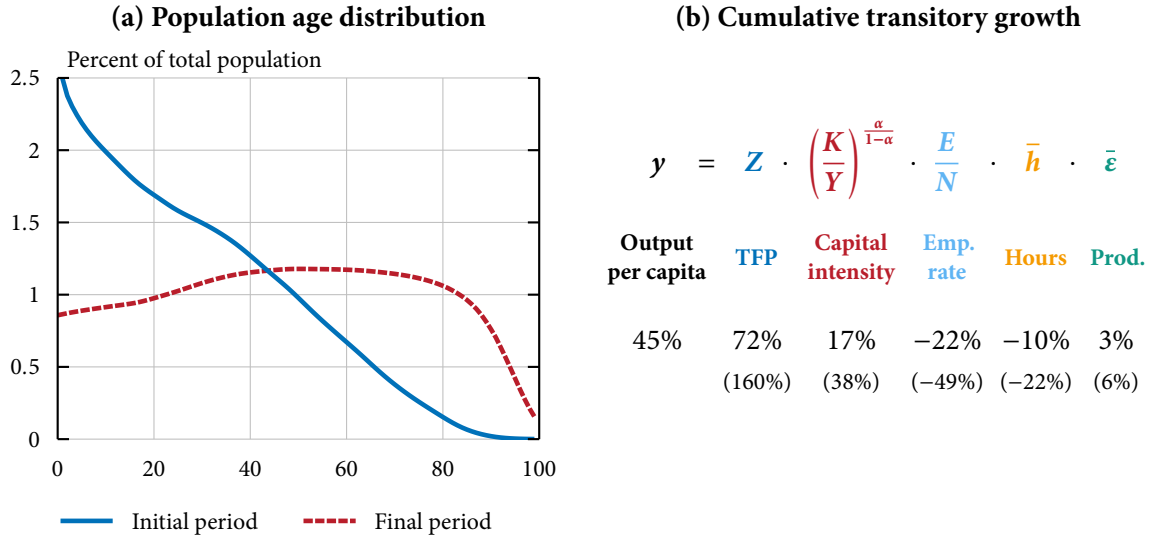


Figure 7. Comparative statics: population structure and output per capita.

Notes. Figure 7b shows the cumulative growth in output per capita and each of its contributing factors along the demographic transition. The numbers in parentheses displays their relative contributions to overall growth.

of semi-endogenous growth introduced in Section 2, which occurs when the demographic transition increases the population size. A simple decomposition of TFP illustrates this point. Specifically, per Definition 2 we can exploit that TFP is proportional to the size of the labour force E raised to γ_Z in steady state. The corresponding proportionality constant provides a measure of the R&D intensity, just as in Equation (4) of the simple model. The scale effect is pinned down by the overall change in employment, E_T/E_0 , where 0 and T denote the initial and final steady states. We then obtain the following:

$$\text{TFP change} = \frac{Z_T}{Z_0} = \frac{Z_T / E_T^{\gamma_Z}}{Z_0 / E_0^{\gamma_Z}} \cdot \left(\frac{E_T}{E_0}\right)^{\gamma_Z} = \text{Change in R\&D intensity} \times \text{Scale effect},$$

$\uparrow 72\%$

$\uparrow 18\%$

$\uparrow 47\%$

where the percentages show the cumulative change in each factor between the initial and final steady states. The 72 percent rise in TFP is due to a 18 percent increase in the R&D intensity and a 47 percent increase in scale.²³ Changes in household behaviour and composition thus impact output per capita through the R&D intensity much like they do through the capital intensity. The difference between TFP and the other factors of production instead lies in the nonrival nature of technology, as seen from the large scale effect.²⁴

6.2 Growth Dynamics

Comparative statics are illustrative but says little about the dynamics during the transition. Therefore, we now turn to the period of interest: 1950 to 2100. Figure 8 plots the growth rate and its corresponding decomposition (that is, the right-hand side of (36)) during this period. Focusing first on the overall growth rate, three key developments emerge. First, the demographic transition positively affects output per capita throughout the second half of the twentieth century, with growth firmly above zero. Then, this effect fades

²³ This decomposition relies on steady-state relationships and therefore has no exact counterpart along the transition path. However, since the exercise already incorporates the full transition from initial to terminal steady state, it still reveals the overall relative importance of the scale and R&D-intensity channels during this transition.

²⁴ A caveat here is that using employment as the relevant scaling variable is somewhat arbitrary. We could just as well use the overall population size, since it grows in parallel with employment in steady state. Switching to the population size (measured in adult equivalents) yields a slightly larger scale effect, 55 percent, although this does not change the basic point of the exercise.

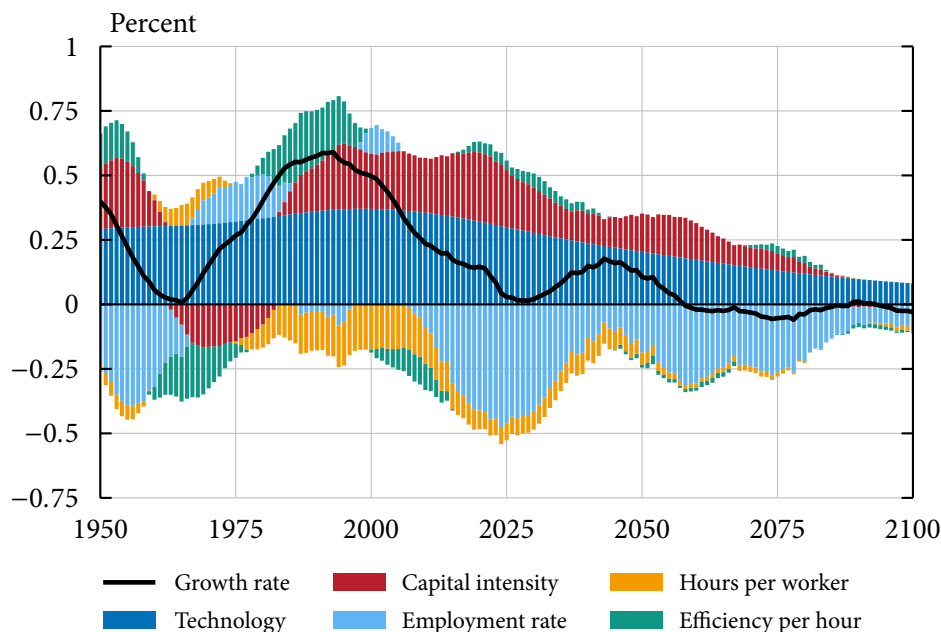


Figure 8. Growth accounting of the baseline model.

after the turn of the century, causing a significant drop in the growth rate. Finally, despite this decline, the demographic development does not negatively affect output per capita. Instead, growth remains around zero throughout the twenty-first century.

Looking at the underlying mechanisms, we again find a qualitative development as predicted in [Section 2](#): the demographic transition generates positive TFP growth and capital deepening, but lowers the employment rate. Moreover, average hours worked declines in periods of positive growth (and vice versa) while average efficiency per hour worked rises in periods where the share of middle-aged workers is higher. The former is due to the parametrisation of household preferences, where the income effect of higher wages on leisure dominates the substitution effect, while the latter follows from the hump shape in workers' life-cycle productivity profile.

[Table 4](#) quantifies these observations by summarising the average annual growth rates of output per capita and its components, first for the entire period considered and then separately for each century. Overall, the demographic transition boosts output per capita by 0.16 percent per year. The effect is primarily driven by the twentieth-century development, where the average annual growth rate is 0.33 percent. By comparison, the observed long-run growth rate of GDP per person in US data is approximately 2 percent per year. From this viewpoint, the impact obtained here is quantitatively significant; taken at face value it implies that over 15 percent of actual US post-war growth can be attributed to transitory demographic factors. This contribution makes demographics comparable in importance to rising educational attainment, whose contribution to US growth over the same period is also around 20 percent according to estimates by Fernald and Jones ([2014](#)).

Just as in the comparative statics, what stands out quantitatively from both [Figure 8](#) and [Table 4](#) is the importance of TFP. Over the full period, TFP grows by 0.25 percent per year, thus accounting for more than 160 percent of economic growth. Its contribution is about two and a half times as large as that of capital deepening. The effect is even larger during the twentieth century, with TFP growing by 0.33 percent per year, about five times faster than the capital intensity, thus driving the bulk of output growth.

Table 4. Growth accounting of the baseline model.

	Output per capita	TFP	Capital intensity	Employment rate	Hours per worker	Efficiency per hour
Period	g_y	g_Z	$\frac{\alpha}{1-\alpha} g_{K/Y}$	$g_{E/N}$	g_h	g_ε
1950–2100	0.16 %	0.25 %	0.11 %	−0.15 %	−0.06 %	0.01 %
1950–2000	0.33 %	0.33 %	0.07 %	−0.05 %	−0.06 %	0.04 %
2001–2100	0.08 %	0.21 %	0.13 %	−0.20 %	−0.05 %	0.00 %
1995	0.55 %	0.37 %	0.25 %	−0.07 %	−0.17 %	0.16 %
2030	0.02 %	0.28 %	0.18 %	−0.41 %	−0.07 %	0.05 %
Difference	−0.53 pp.	−0.09 pp.	−0.08 pp.	−0.34 pp.	0.10 pp.	−0.11 pp.

Notes. The table reports average annual growth rates according to the growth decomposition in Equation (36). Individual growth rates may not sum to totals due to rounding.

A declining share of workers in the population constitutes the main drag on output growth, depressing the annual growth rate by on average 0.15 percentage points overall and by 0.20 percentage points during the twenty-first century. This happens as the average retirement age does not keep up with increasing life expectancy; recall Figure 5b, which shows that although there is a small shift towards later retirement over time, the increase in the retirement age is too marginal to meaningfully increase the gap between the share of retired workers in the population and the share of people aged 65 and above.

Zooming in on the growth decline from peak in 1995 to trough in 2030, the lower half of Table 4 shows that demographic change generates a 0.53 percentage point drop in the growth rate over these three decades. The majority—65 percent—comes from a 0.34 percentage point decline in employment rate growth as the baby boom retires, with smaller declines around 0.1 percentage points in TFP, capital intensity, and average efficiency growth contributing the remainder. Demographics therefore exert significant downward pressure on contemporary growth rates. However, this decline should not be interpreted as evidence that population ageing is particularly bad for economic output. Rather, demographic change generates a long period of above-normal growth in the last quarter of the twentieth century, when baby boomers were of prime working age, and we are just now experiencing the end of that period as these generations retire. As stressed in the simple model in Section 2, this reversion would occur even without rising life expectancy.

6.3 Is It All About Boomers? Identifying the Demographic Effects

Related to the last point above, it appears at first sight that Figure 8 just reflects the baby boom dynamics predicted in Section 2. The share of young workers increases when the baby boomers enter adulthood in the 1960s and 1970s, which improves the employment rate and worsens the capital intensity and average productivity. When these cohorts become middle-aged in the 1980s and 1990s, growth through TFP, the capital intensity, and average productivity improves. Finally, they retire in the early decades of the twenty-first century, thereby causing a negative growth impact via the employment rate. Yet, Section 2 also stresses the increase in life expectancy from rising middle- and old-age survival rates. Which of these forces, if any, is the key driver of the results above?

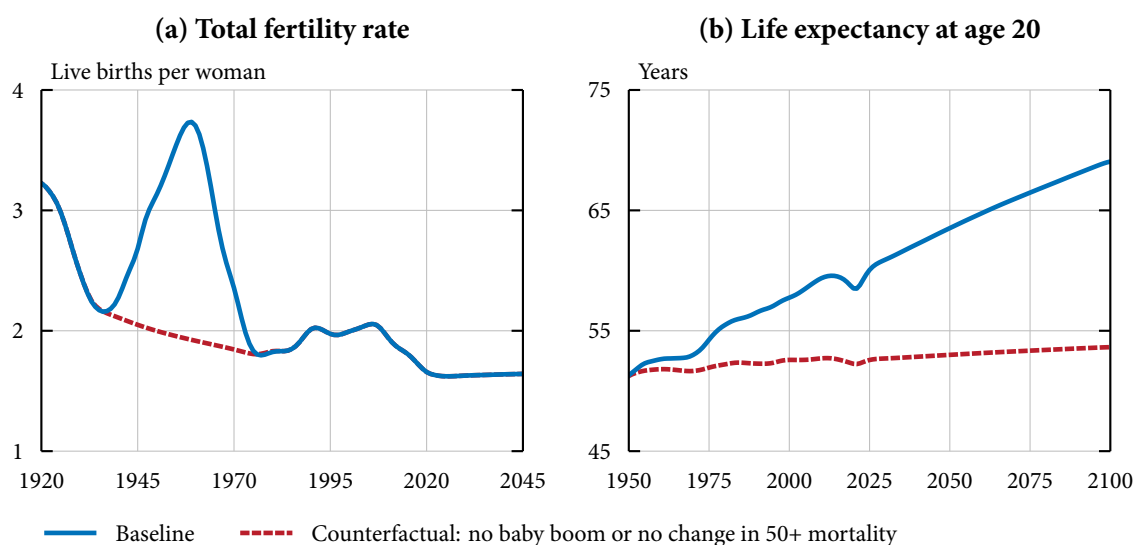


Figure 9. Demographic counterfactual scenarios.

Notes. The total fertility rate and the life expectancy are computed under the assumptions that the fraction of women in an age group and the average fraction of the year lived at the age of death remain the same as in official data and projections.

To answer this question, I consider three counterfactual scenarios. First, I simulate the model under the assumption that neither the baby boom nor the rise in survival rates happens. That is, I replace fertility rates between 1935 and 1975 with interpolated values and hold survival rates above the age of 50 fixed from 1950 onward. As shown in Figure 9, these adjustments remove the hump shape in fertility associated with the baby boom and almost all life expectancy improvements after 1950. I then switch either fertility or mortality back to the baseline calibration and look at the change relative to the baseline model. Without strong interaction effects, these scenarios isolate the impacts of the baby boom, of the change in middle- and old-age mortality, and of all other demographic changes.

Figure 10 plots the resulting decomposition of the cumulative change in output per capita into each demographic factor. While the growth swings in Figure 8 are clearly attributed to the life-cycle phases of the baby boomers, they are by no means the sole explanation behind the overall development. Between 1950 and 2100, output per capita rises by a total of 27.5 percent, which is accounted for by an 8.2 percent increase due to the baby boom, a 3.0 percent increase due to middle- and old-age mortality changes, a 14.7 percent increase due to other demographic factors, and a negligible residual reflecting interactions between the three. The baby boom and changing mortality consequently explain around half of the baseline results, with remaining demographic changes accounting for the rest.

Table 5 displays the demographic contributions to each component of the average growth rate between 1950 and 2100. This shows that the baby boom and changing mortality generate around a quarter each of the total technological progress. Transition dynamics notwithstanding, this TFP contribution (and the subsequent income effect on hours worked) is the baby boom's only impact in the end, thus confirming the prediction from the simple model. This is unsurprising since the counterfactual fertility rates affect the cohorts that become adults after 1955 and die with certainty no later than 2075. Before and after these years, the age structure of the adult population is nearly identical to the baseline. Most changes in the capital intensity and the employment rate are instead driven by life expectancy improvements. Despite its positive impact on output per capita, longer lifespans also increase hours per worker, reflecting households' need to finance longer retirement.

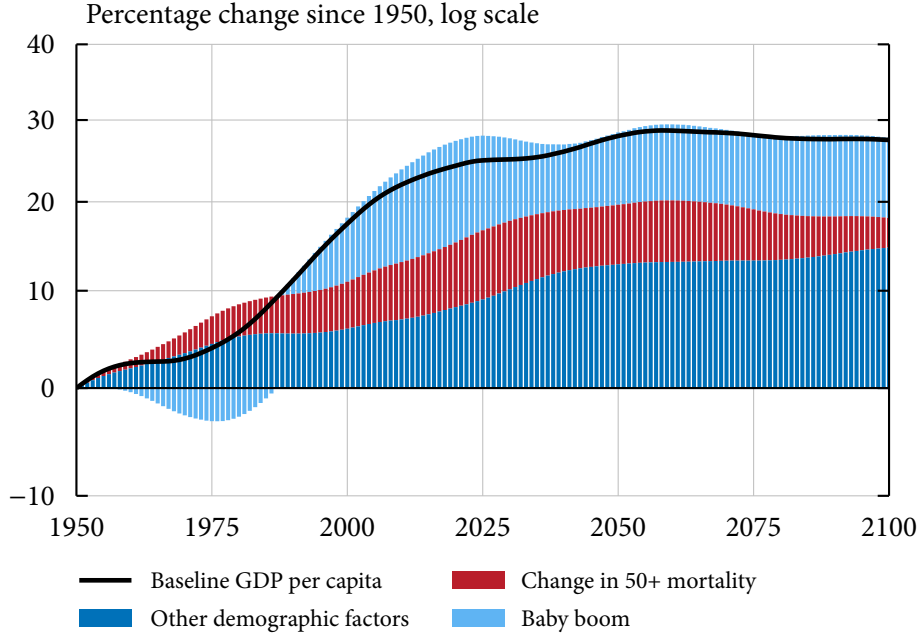


Figure 10. Identifying the impact of the demographic forces.

What explains the residual factor? First, the population in 1950 is not stationary. Changes in the population structure therefore occur even without further changes in the underlying demographic variables, thus impacting the saving rate, the employment rate, and the average productivity per hour worked. This shift in the age structure is largely complete by the end of the twentieth century. Additionally, the changes in fertility, mortality, and migration rates that nevertheless occur raise the population size, so the scale effect on TFP is also at work here, as seen from the large impact on TFP growth.

6.4 Is It All About Technological Progress? The Exogenous Growth Case

The finding that the demographic transition does not negatively impact per-capita output contrasts with the general notion of population ageing as a major drag on economic activity. For instance, Krueger and Ludwig (2007), the perhaps closest paper to the analysis here, use a similar quantitative model and find a 12.6 percent cumulative drop in US output per capita between 2005 and 2080 due to the demographic transition. However, most previous work, including Krueger and Ludwig, ignore the TFP channel. Another important question is thus to what extent this additional mechanism explains the difference.

As a final exercise, I therefore analyse a version of the model without technological change. Specifically, the benchmark model nests a standard model without endogenous growth as the special case with perfect substitution between intermediate firms ($\rho = 1$) and a zero intermediate-firm exit rate ($\delta_z = 0$). The former eliminates profits, thus forcing the patent price and R&D investment to zero, which in turn reduces the intermediate-firm dynamics to $z_{t+1} = (1 - \delta_z)z_t$. The latter ensures that the measure of intermediate firms remains constant over time. I also recalibrate the structural parameters β , ψ , Ω_A , and δ_k here to maintain the baseline calibration targets.

Figure 11a plots the growth rate under this specification against that of the baseline model. The positive effects now disappear, with growth rates consistently around 0.1 to 0.4 percentage points below the baseline. The cumulative decline in output per capita is nearly identical to Krueger and Ludwig (2007): 12.1 percent between 2005 and 2080 and 12.2 percent over the full period. In comparison, for the same periods the

Table 5. Growth decomposition of the demographic forces.

	Output per capita	TFP	Capital intensity	Employment rate	Hours per worker	Efficiency per hour
Period	g_y	g_Z	$\frac{\alpha}{1-\alpha} g_{K/Y}$	$g_{E/N}$	g_h	g_ε
Average growth rate	0.16 %	0.25 %	0.11 %	−0.15 %	−0.06 %	0.01 %
Baby boom	0.05 pp.	0.07 pp.	0.00 pp.	0.00 pp.	−0.02 pp.	0.00 pp.
50+ mortality	0.02 pp.	0.06 pp.	0.05 pp.	−0.12 pp.	0.03 pp.	0.00 pp.
Other changes	0.09 pp.	0.13 pp.	0.05 pp.	−0.04 pp.	−0.07 pp.	0.01 pp.

Notes. The table reports average annual growth rates between 1950 and 2100 according to the growth decomposition in [Equation \(36\)](#) and the percentage point contributions by each demographic factor. Individual growth rates may not sum to totals due to rounding.

baseline model exhibits positive cumulative growth of 6.3 and 27.5 percent, respectively. The difference is driven almost entirely by the TFP difference, as shown in [Figure 11b](#). The impacts on the capital intensity, the employment rate, and the average efficiency are virtually unchanged.²⁵ There is also a small counteracting effect in hours worked: lower income leads to a rise in hours, and this raises output. But this effect is too small to offset the lack of technological progress. Thus, whether the demographic transition raises or lowers output per capita turns out to depend crucially on whether or not we account for its impact on technological change.

7 The Importance of the R&D Production Function

A central element in the results above is the R&D process, and this section closes the paper by examining the model's sensitivity to two critical components: the knowledge spillover parameter ϕ and the specification of the R&D production function. The former is particularly important, since ϕ determines the extent to which R&D becomes easier or harder over time, and its true value is likely subject to considerable uncertainty due to the challenges in measuring productivity growth and R&D investment. Additional robustness checks, detailed in [Online Appendix E](#), include (i) gradually raising the normal retirement age as scheduled by the Social Security Administration, (ii) removing idiosyncratic income risk, (iii) eliminating the bequest motive, (iv) adopting a flat income tax, (v) imposing log preferences on consumption to fix long-run hours worked, and (vi) introducing a part-time wage penalty on hours worked. None of these adjustments meaningfully alter the benchmark results.

To assess the role of knowledge spillovers, I vary ϕ to ensure that the growth exponent γ_y aligns with the upper and lower bounds identified by Jones (2002). For weaker spillovers, I set $\phi = -1.4$, for which γ_y equals 0.07. This is also the value of ϕ found by Bloom *et al.* (2020) for the aggregate US economy when $\lambda = 0.75$. For stronger spillovers, I set $\phi = 0.4$, which gives $\gamma_y = 0.36$. To evaluate the specification of the R&D production function, I consider an alternative commonly used in the growth literature, where labour rather than final goods serves as R&D input:

$$z_{t+1} = (1 - \delta_z)z_t + \nu L_{zt}^\lambda z_t^\phi,$$

²⁵ See [Figure F7](#) in [Online Appendix F](#), which fully decomposes the growth rate gap.

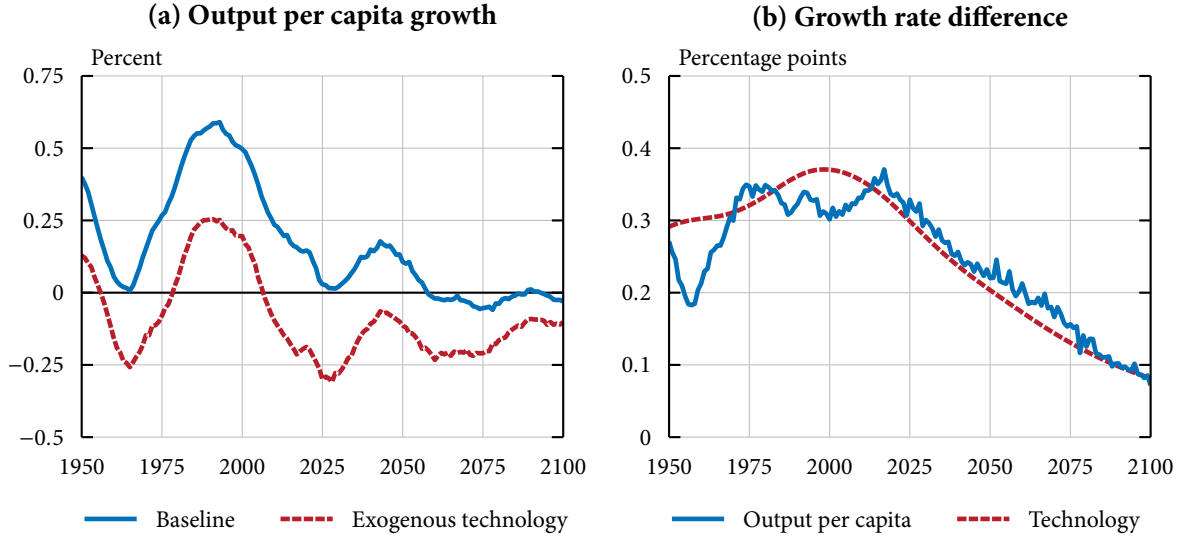


Figure 11. Comparing growth with and without endogenous technological change.

where L_{zt} represents total R&D labour.²⁶ For comparability, I set $\phi = 0.13$ to match the long-run growth rate of the baseline model. In all cases, I again re-estimate the internally calibrated parameters to maintain consistency with the baseline calibration targets.

Figure 12 displays the growth rates from these alternatives against that of the baseline. Qualitatively, changing the knowledge spillover is straightforward: the larger the knowledge spillover, the larger the growth impact from R&D, since researchers become more productive the more knowledge is created. Figure 12 confirms this prediction. Changing the R&D input from final goods to labour generates higher relative demand for labour from producers, with subsequent general equilibrium effects on factor prices. Since the R&D sector is a small part of the economy, these effects are minor. None of the key mechanisms change here, as the cost of R&D (the wages of researchers) and the patent purchases by intermediate firms are still financed via household savings. The impact relative to the baseline is therefore negligible.

Growth rates are overall positive across the board (albeit quantitatively insignificant with small knowledge spillovers). Relative to the baseline, the average annual growth rate between 1950 and 2100 decreases and increases by about 0.1 to 0.15 percentage points with the different degrees of knowledge spillovers and remains unaffected when labour is the sole R&D input. Only in the most pessimistic calibration do we observe a negative impact on per-capita output during the twenty-first century, but even here the impact is small at about -0.02 percent per year. Therefore, although reasonable variations of the R&D process impact the quantitative findings, they do not change the basic point of this paper: that current demographic trends improves output per capita under endogenous technological change.

8 Conclusion

Neither the comparative statics nor the transition dynamics in this paper suggest that current demographic change is inherently detrimental to economic growth. If anything, the effects are (temporarily) positive

²⁶ The labour and goods market conditions also become $L_t + L_{zt} = \sum_j N_{jt} \int_X \ell_{jt}(x) d\Phi_{jt}$ and $Y_t + A_{t+1}^M = C_t + G_t + [K_{t+1} - (1 - \delta_k)K_t]$. As shown in [Online Appendix A](#), long-run growth again follows the population growth rate via $1 + g_z = (1 + n)^\chi$, but now with $\chi \equiv \frac{\lambda}{1 - \phi - \lambda \frac{\theta(1-\sigma)}{1+\theta\sigma} \frac{\alpha}{1-\alpha} \frac{1-\rho}{\rho}}$.

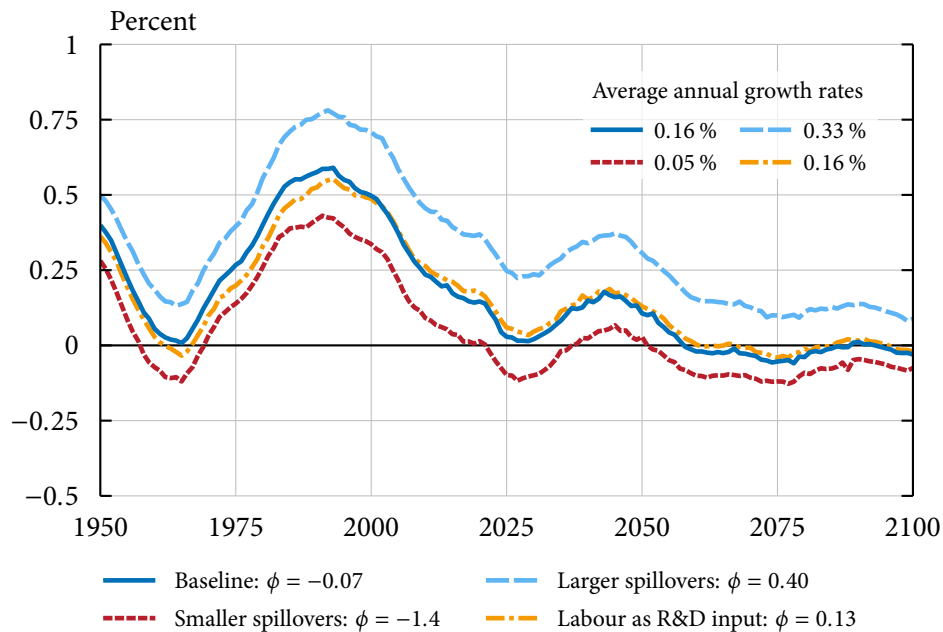


Figure 12. Growth rates with different degrees of knowledge spillovers and R&D inputs.

and quantitatively large, at least on par with the contribution of rising educational attainment to US growth during the second half of the twentieth century. While translating these growth effects into welfare conclusions is beyond the scope of the paper, the results suggest that the macroeconomic consequences of demographic change may be considerably more favourable than standard analyses imply. These conclusions are also likely conservative, since several adjustment margins abstracted from here—such as human capital accumulation, open-economy aspects, and R&D subsidies—would plausibly amplify rather than attenuate the central mechanism.

Ultimately, the paper’s contribution lies less in quantifying the exact growth impact of demographic change than in highlighting that technological progress, operating through higher savings and R&D investment in ageing societies, appears to be a key factor in understanding its macroeconomic implications. This observation raises natural questions for related literatures built on exogenous growth models, including analyses of social security reform, migration policy, fiscal policy reform, and optimal taxation. Jones (2022b), for instance, shows that endogenous technological change can substantially alter optimal income tax progressivity. The insights here provide a natural starting point for further work on these questions.

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Endogenous Technological Change Along the Demographic Transition Online Supplement

Markus Pettersson[†]

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This supplement provides (a) derivations of the balanced growth rates in the main model; (b) details on the estimation of the life-cycle productivity profile; (c) details on the calibration of taxes; (d) details on the internally calibrated parameters; (e) additional sensitivity analyses; and (f) additional figures.

Appendix A Deriving the Stationary Growth Rates

The stationary growth rates of the benchmark model and of the version with labour as the only R&D input are straightforwardly derived following Jones (1995) and Boppart and Krusell (2020).

A.1 TFP, Output Per Capita, Hours Per Worker

In a stationary equilibrium, interior solutions to the household problem are characterised by the Euler equation, the intratemporal first-order condition for hours worked, and the budget constraint:

$$\frac{\Omega_{jc}}{1 + \tau^c} c_j^{-\sigma} = \left(1 + r(1 - \tau^k)\right) \left\{ (1 - s_j) \Omega_a (a_{j+1}^b)^{-\sigma} + s_j \beta \frac{\Omega_{j+1,c}}{1 + \tau^c} \mathbb{E} \left[c_{j+1}^{-\sigma} \mid \eta \right] \right\}, \quad (\text{A.1})$$

$$\psi h_j^{1/\theta} = \Omega_{jc} \frac{(1 - \tau^{wm}(w\ell_j) - \tau^b) w \varepsilon_j \eta}{c_j^\sigma (1 + \tau^c)}, \quad (\text{A.2})$$

$$a_{j+1} + (1 + \tau^c) c_j = (1 + r(1 - \tau^k)) a_j + (1 - \tau^w(w\ell_j) - \tau^b) w \ell_j + tr + b(R_j), \quad (\text{A.3})$$

where $\tau^{wm}(e) \equiv \frac{\partial}{\partial e} (\tau^w(e)e)$ in (A.2) denotes the marginal income tax rate. Let g_x denote the stationary growth rate of a variable x . By the Euler equation, constant growth of consumption requires a constant interest rate. Wages are standard neoclassical, so in a stationary equilibrium with a constant interest rate and capital-output ratio, wages grow with the rate of TFP. For the first-order condition (A.2) to hold along a balanced growth path, we then necessarily need $(1 + g_h)^{1/\theta} = (1 + g_Z)(1 + g_c)^{-\sigma}$. Likewise, the budget constraint (A.3) is only consistent with balanced growth if consumption grows with the rate of output per capita and labour income: $1 + g_c = 1 + g_y = (1 + g_Z)(1 + g_h)$. Meanwhile, TFP is just the measure of intermediate firms raised to a power, $Z = z^{\frac{\alpha}{1-\alpha} \frac{1-\rho}{\rho}}$, so its growth rate is similarly the growth rate of

[†] Department of Economics, Stockholm University. Email: markus.pettersson@su.se.

intermediate firms raised to the same power. Combining these conditions yields that the growth rates of TFP, output per capita, and hours per worker must satisfy

$$1 + g_Z = (1 + g_z)^{\frac{\alpha}{1-\alpha} \frac{1-\rho}{\rho}}, \quad 1 + g_y = (1 + g_Z)^{\frac{1+\theta}{1+\theta\sigma}}, \quad 1 + g_h = (1 + g_Z)^{\frac{\theta(1-\sigma)}{1+\theta\sigma}}, \quad (\text{A.4})$$

and substituting $1 + g_z = (1 + n)^\chi$ for some χ into [Equation \(A.4\)](#) gives the growth rates in [Definition 2](#).

A.2 Intermediate-Firm Growth Rate: Baseline Model

Next, the intermediate-firm growth rate is given by $1 + g_z = 1 - \delta_z + \nu Q^\lambda z^{\phi-1}$, and this is constant if and only if the last term on the right-hand side is constant. The latter holds only if

$$(1 + g_z)^{1-\phi} = (1 + g_Q)^\lambda. \quad (\text{A.5})$$

The growth rate of R&D investment equals the aggregate output growth rate $(1 + g_Z)(1 + g_L)$ by the goods market condition. Employment grows by the rate of the population $1 + n$ whereas labour productivity is constant for a fixed population age structure. The labour force growth rate is therefore given by $1 + g_L = (1 + g_h)(1 + n)$. Together with [Equation \(A.4\)](#), this allows us to rewrite the growth rate of R&D investment into

$$1 + g_Q = (1 + g_Z)(1 + g_h)(1 + n) = (1 + g_z)^{\frac{1+\theta}{1+\theta\sigma} \frac{\alpha}{1-\alpha} \frac{1-\rho}{\rho}} (1 + n).$$

Plugging this into [\(A.5\)](#) and rearranging terms yields the growth rate in [Definition 2](#) in the paper:

$$1 + g_z = (1 + n)^\chi, \quad \text{where} \quad \chi \equiv \frac{\lambda}{1 - \phi - \lambda \frac{1+\theta}{1+\theta\sigma} \frac{\alpha}{1-\alpha} \frac{1-\rho}{\rho}}. \quad (\text{A.6})$$

A.3 Intermediate-Firm Growth Rate: R&D With Only Labour

If R&D uses only labour, then the R&D process is given by $1 + g_z = 1 - \delta_z + \nu L_z^\lambda z^{\phi-1}$. In steady state, R&D labour L_z must grow by the rate of total labour supply according to the labour market condition. Again inspecting the right-hand side, we thus get a constant growth rate $1 + g_z$ if and only if

$$(1 + g_z)^{1-\phi} = (1 + g_L)^\lambda. \quad (\text{A.7})$$

Using [Equation \(A.4\)](#), we can rewrite the labour force growth rate into

$$1 + g_L = (1 + g_h)(1 + n) = (1 + g_z)^{\frac{\theta(1-\sigma)}{1+\theta\sigma} \frac{\alpha}{1-\alpha} \frac{1-\rho}{\rho}} (1 + n).$$

Plugging this into [\(A.7\)](#) and rearranging terms yields the growth rate

$$1 + g_z = (1 + n)^\chi, \quad \text{where} \quad \chi \equiv \frac{\lambda}{1 - \phi - \lambda \frac{\theta(1-\sigma)}{1+\theta\sigma} \frac{\alpha}{1-\alpha} \frac{1-\rho}{\rho}}. \quad (\text{A.8})$$

Note that TFP growth collapses to the benchmark growth rate in Jones (1995) if we consider a steady state with constant hours worked (via log preferences, $\sigma \rightarrow 1$) and a substitution parameter for intermediates that exactly equals the capital share parameter ($\rho = \alpha$).

Appendix B Estimating the Life-Cycle Earnings Profile

I parametrise the age-efficiency profile $\{\varepsilon_j\}_{j=1}^J$ as the exponential of a quadratic age polynomial: $\varepsilon_j = \exp \{\vartheta_0 + \vartheta_1 j + \vartheta_2 j^2\}$. In the model, the hourly wage of an individual i of age j at time t is given by $w_{ijt} = w_t \varepsilon_j \eta_{ijt}$, where w_t is a common wage trend and η_{ijt} captures any idiosyncratic differences. This motivates the fixed effects regression

$$\ln w_{ijt} = \varrho_t + \varrho_i + \vartheta_0 + \vartheta_1 j + \vartheta_2 j^2 + u_{ijt}, \quad (\text{B.1})$$

where ϱ_t is a time fixed effect, ϱ_i is an individual fixed effect, and u_{ijt} is an error term. Equation (B.1) implicitly captures cohort effects through the individual fixed effects and it is well known that collinearity between age, time, and cohorts prohibits simultaneous identification of these effects. As a partial remedy, I use the approach advocated by Heckman and Robb (1985) and replace the time fixed effect by two macroeconomic variables which plausibly proxy for the underlying unobserved time variables in the context of an earnings regression: log of the aggregate real wage level and the percentage point deviation of the unemployment rate from its long-run mean. The former corresponds to $\ln w_t$ and controls for secular wage growth and the latter (which is also used by French, 2005) controls for fluctuations in the business cycle.

I estimate Equation (B.1) with micro data on earnings from the nationally representative SRC sample of the Panel Study of Income Dynamics (PSID) for survey years 1968 to 2019 (which correspond to calendar years 1967 to 2018). Individual wages are imputed as total annual labour income divided by annual hours worked. The aggregate wage used to proxy the time fixed effect is obtained from the national accounts by dividing total private industry wages (BEA NIPA Table 2.3) by total private industry hours worked (BEA NIPA Table 6.9). The unemployment rate is taken from the Bureau of Labor Statistics (BLS, series ID LNS14000000). All nominal variables are deflated into 2012 dollars using the PCE price index (BEA NIPA Table 2.3.4).

For the benchmark estimation, I impose standard sample restrictions (see for instance French, 2005, Heathcote, Storesletten and Violante, 2010, and Huggett, Ventura and Yaron, 2011): I select male household heads with no inconsistencies in reported age, who work between 300 and 5,840 hours a year (30 percent of part time and twice full time, respectively), and whose hourly wage exceeds \$3 per hour and does not exceed \$100 per hour in 2012 dollars. I consider individuals between the ages of 18 and 75. This goes against the standard practice of excluding ages at the beginning and end of the working life to avoid sample selection issues relating to work-life entry and exit. This choice is motivated by the need for an efficiency profile for all ages above 20, given that retirement in the model is endogenous. The alternative, estimating the age profile on individuals between, say, the ages of 25 and 60, instead requires extrapolation of the age profile to younger and older ages, and it is not clear that this approach is preferable. An upper bound at 75 is nevertheless imposed to ensure there are at least 100 observations in each age group. Extrapolation beyond 75 is inconsequential, since between 95 and 99 percent of model households retire before 75. The final sample consists of 90,832 person-year observations.

Table B.1 shows the estimation results along with several robustness checks. Column (1) corresponds to the age profile in Figure F.5a in Appendix F. Column (2) shows standard OLS estimates and column (3) includes only individual fixed effects. In both cases, secular wage growth is interpreted as life-cycle earnings differences, and this generates steeper profiles; productivity at peak age is 115 to 130 percent larger than the initial age, compared to 70 percent for the main estimation. This underlines the importance

Table B.1. Estimation of deterministic age-efficiency profile.

	Benchmark	Robustness checks					
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Constant	−1.3734*** (0.2250)	0.9857*** (0.0191)	1.1419*** (0.0365)	−0.9102*** (0.1783)	−1.2920*** (0.2043)	−1.0902*** (0.1655)	−1.1382*** (0.1045)
Age	0.0734*** (0.0020)	0.0954*** (0.0010)	0.0835*** (0.0018)	0.0813*** (0.0018)	0.0709*** (0.0018)	0.0610*** (0.0015)	0.0606*** (0.0010)
Age squared	−0.0008*** (0.0000)	−0.0010*** (0.0000)	−0.0008*** (0.0000)	−0.0008*** (0.0000)	−0.0008*** (0.0000)	−0.0007*** (0.0000)	−0.0007*** (0.0000)
Individual FE	✓		✓	✓	✓	✓	✓
Time controls ^a	✓			✓	✓	✓	✓
Aggregate wage from	BEA			BLS	BEA	BEA	BEA
Female heads					✓	✓	✓
Spouses/partners						✓	✓
Additional controls ^b							✓
Observations	90,832	90,832	90,832	90,832	110,169	165,034	161,012
Adjusted R^2	0.160	0.121	0.156	0.162	0.158	0.152	0.153

Notes. Dependent variable: log real hourly wage. Wages defined as labour earnings/hours. Regressors of interest: quadratic age polynomial. Robust standard errors in parentheses. *, **, and *** denote statistical significance at the 10 percent, 5 percent, and 1 percent levels, respectively.

^a Includes controls for the aggregate real wage level (using BEA or BLS data) and business cycle fluctuations (as proxied by the deviation of the unemployment rate from its long-run mean).

^b Includes controls for education level (eight groups), family size, marital status (married, single, widowed, divorced, separated), and fixed effects for place of residence (50 states and D.C. plus abroad).

of controlling for time effects. Column (4) changes the aggregate wage measure to average hourly earnings of production and nonsupervisory employees (BLS, series ID CES0500000008). This wage exhibits lower growth in recent decades than the imputed wage from the BEA, and the estimation results are therefore similar to columns (2) and (3).¹ Columns (5) to (7) expand the sample to include spouses, partners, and female household heads and add additional individual-level controls that may change over time. These additions lower the point estimates somewhat, although these samples raise additional concerns for sample selection and also provide worse fits to the data as measured by the adjusted R^2 .

Lastly, for the model scenario in which wages are an increasing function of hours worked in [Appendix E](#), I run identical estimations to above with the only exception that individual wages are constructed as (total annual labour income)/(annual hours worked)^{1.415} in the PSID data. [Table B.2](#) shows the results from these regressions.

Appendix C Constructing Tax Rates

This section explains the calibration of the model taxes. First, I describe the construction of the aggregate tax rates from the national accounts, then the estimation a progressive income tax function, and finally the calibration of the income tax function used in the model (which builds on the former two).

¹ I use the BEA wage measure as the benchmark since the BLS wage is more limited in scope.

Table B.2. Estimation of deterministic age-efficiency profile with part-time wage penalty.

	Benchmark	Robustness checks					
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Constant	−4.1083*** (0.2291)	−1.8276*** (0.0200)	−1.6727*** (0.0366)	−3.6135*** (0.1809)	−4.0415*** (0.2084)	−3.9382*** (0.1669)	−3.9195*** (0.1121)
Age	0.0540*** (0.0020)	0.0757*** (0.0010)	0.0634*** (0.0018)	0.0616*** (0.0018)	0.0515*** (0.0018)	0.0438*** (0.0014)	0.0400*** (0.0010)
Age squared	−0.0006*** (0.0000)	−0.0008*** (0.0000)	−0.0006*** (0.0000)	−0.0006*** (0.0000)	−0.0006*** (0.0000)	−0.0005*** (0.0000)	−0.0004*** (0.0000)
Individual FE	✓		✓	✓	✓	✓	✓
Time controls ^a	✓			✓	✓	✓	✓
Aggregate wage from	BEA			BLS	BEA	BEA	BEA
Female heads					✓	✓	✓
Spouses/partners						✓	✓
Additional controls ^b							✓
Observations	90,832	90,832	90,832	90,832	110,169	165,034	161,012
Adjusted R ²	0.123	0.094	0.120	0.125	0.120	0.110	0.113

Notes. Dependent variable: log real hourly wage. Wages defined as labour earnings/hours^{1.415}. Regressors of interest: quadratic age polynomial. Robust standard errors in parentheses. *, **, and *** denote statistical significance at the 10 percent, 5 percent, and 1 percent levels, respectively.

^a Includes controls for the aggregate real wage level (using BEA or BLS data) and business cycle fluctuations (as proxied by the deviation of the unemployment rate from its long-run mean).

^b Includes controls for education level (eight groups), family size, marital status (married, single, widowed, divorced, separated), and fixed effects for place of residence (50 states and D.C. plus abroad).

C.1 Aggregate Tax Rates

The methodology to construct the aggregate tax rates on consumption, capital, and labour income is taken off-the-shelf from Fernández-Villaverde *et al.* (2015), which in turn builds on Jones (2002) and Mendoza, Razin and Tesar (1994). In short, each tax rate is given by aggregating all relevant tax revenues at the general government level and then dividing by the corresponding tax base. All data for this exercise are taken from BEA NIPA tables. Table C.1 summarises the variables that I use.

The average consumption tax rate τ^c is given by

$$\tau^c = \frac{TPI - PRT}{C - (TPI - PRT)}. \quad (C.1)$$

The numerator of (C.1) is the revenue from consumption taxation. I subtract property taxes from total taxes on production because homeowners in the national accounts are treated as businesses that rent their properties to themselves. Property taxes are therefore incorporated as taxes on capital instead. The consumption tax base in the denominator is total personal consumption expenditures net of consumption taxes paid (that is, the pre-tax value of consumption).

Table C.1. Tax variables in the national accounts.

Variable	Explanation	Source
<i>C</i>	Personal consumption expenditures	BEA NIPA Table 1.1.5 line 2
<i>EC</i>	Compensation of employees	BEA NIPA Table 1.12 line 2
<i>W</i>	Wages and salaries	BEA NIPA Table 1.12 line 3
<i>PRI</i>	Proprietors' income ^a	BEA NIPA Table 1.12 line 9
<i>RI</i>	Rental income of persons ^a	BEA NIPA Table 1.12 line 12
<i>CP</i>	Corporate profits ^a	BEA NIPA Table 1.12 line 13
<i>NI</i>	Net interest and miscellaneous payments	BEA NIPA Table 1.12 line 18
<i>PCT</i>	Personal current taxes	BEA NIPA Table 3.1 line 3
<i>TPI</i>	Taxes on production and imports	BEA NIPA Table 3.1 line 4
<i>CT</i>	Taxes on corporate income	BEA NIPA Table 3.1 line 5
<i>CSI</i>	Contributions for government social insurance	BEA NIPA Table 3.1 line 7
<i>PRT</i>	Property taxes	BEA NIPA Table 3.3 line 9
<i>LABSH</i>	Labour share of national income	BEA NIPA Table 1.12 ^b

^a With inventory valuation adjustment and capital consumption adjustment.

^b See [Appendix D](#) for details.

The NIPA tables do not provide a breakdown of personal current taxes into labour and capital income. To make this split, I construct an average personal income tax rate τ^p as an intermediate step via

$$\tau^p = \frac{PCT}{W + LABSH \cdot PRI + CI}, \quad \text{where} \quad CI \equiv (1 - LABSH)PRI + RI + CP + NI.$$

The numerator is the sum of personal current taxes at the federal, state and local levels. The tax base is the sum of wages, proprietors' income, and capital income (*CI*). Here, proprietors' income is assumed to be split between labour and capital income in the same proportion as overall national income, which is consistent with the construction of the labour share in [Appendix D](#).

I then estimate the total revenue from personal taxes on income and capital as $\tau^p(W + LABSH \cdot PRI)$ and $\tau^p CI$, respectively. The average labour income and capital tax rates are subsequently given by

$$\tau^w = \tau^p \frac{W + LABSH \cdot PRI}{EC + LABSH \cdot PRI} \quad \text{and} \quad \tau^k = \frac{\tau^p CI + CT + PRT}{CI + PRT}. \quad (C.2)$$

While not used directly in the benchmark model, one may also construct a social security tax rate as

$$\tau^b = \frac{CSI}{EC + LABSH \cdot PRI}. \quad (C.3)$$

The sum of τ^w and τ^b gives the average labour income tax rate used by Fernández-Villaverde *et al.* (2015). [Figure C.1](#) plots the estimated tax rates, which highlights that imposing constant tax rates in the model for consumption, labour income and capital is a reasonable assumption.

C.2 Estimating the Income Tax Rate Function

To estimate the income tax function, I compute average tax rates at hypothetical levels of income and fit [Equation \(35\)](#) in the main paper to these synthetic data. This process follows the methodology of the

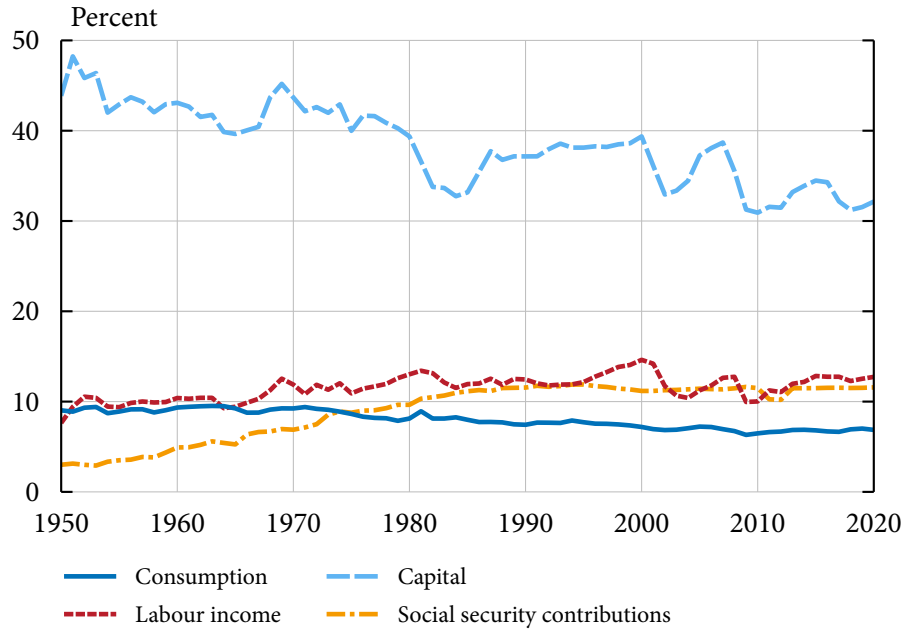


Figure C.1. Aggregate tax rate estimates from the national accounts.

OECD tax database for the United States, which creates effective tax rates by applying applicable tax rules and regulations for given years and earnings levels and then dividing the resulting net tax liabilities by gross earnings. These calculations include taxation at all levels of government for a household assumed to live in Detroit, Michigan.

For simplicity, I only consider single households without children, in line with the primary estimates published by the OECD.² This choice is of secondary importance since I eventually scale the estimated tax function to match the national accounts. I also abstract from social security contributions since those are modelled separately in my framework. The subsections below outline the formulas and parameters for this particular case.³

C.2.1 Taxable Earnings

Taxable earnings at government level $x \in \{\text{fed, state, local}\}$ is given by gross income GI minus a tax allowance TAXALLOW_x , provided that this is positive:

$$e^x(\text{GI}) = \max\{\text{GI} - \text{TAXALLOW}_x, 0\}.$$

At the federal level, the allowance consists of a standard deduction STDALLOW and a personal exemption $\text{EXEMPT}_{\text{fed}}$. The personal exemption is reduced at a taper rate ϕ_{ex}^T for every USD 2,500 that gross income exceeds the threshold THOLD_{ex} . At the state and local levels, the allowances are fixed personal exemptions $\text{EXEMPT}_{\text{state}}$ and $\text{EXEMPT}_{\text{local}}$, respectively. Thus,

$$\text{TAXALLOW}_{\text{fed}} = \text{STDALLOW} + \text{EXEMPT}_{\text{fed}} \left(1 - \phi_{\text{ex}}^T \left[\frac{\max\{\text{GI} - \text{THOLD}_{\text{ex}}, 0\}}{2500} \right] \right),$$

² See Table I.5, available for download at [OECD.Stat](#).

³ The implementation code (available upon request) also incorporates different household compositions (with respect to children and marital status) and social security contributions. Supplementary documentation for these cases and the parameter values used are given in the OECD publication *Taxing Wages* available at the [OECD iLibrary](#).

$$\text{TAXALLOW}_{\text{state}} = \text{EXEMPT}_{\text{state}},$$

$$\text{TAXALLOW}_{\text{local}} = \text{EXEMPT}_{\text{local}},$$

where $\lceil \cdot \rceil$ is the ceiling function: $\lceil x \rceil = \min\{n \in \mathbb{Z} : n \geq x\}$.

C.2.2 Taxes

Taxable earnings at the state and local levels are subject to flat tax rates τ^{state} and τ^{local} . The federal income tax is progressive, with higher marginal tax rates at higher levels of income. Consider N federal tax brackets with marginal tax rates $\tau_1^{\text{fed}}, \dots, \tau_N^{\text{fed}}$ starting at earnings thresholds $\bar{e}_1, \dots, \bar{e}_N$, where $\bar{e}_1 = 0$. Given a largest applicable federal tax bracket $I = \max\{i : e^{\text{fed}}(\text{GI}) > \bar{e}_i\}$, the tax liability at each level of government is then given by functions $T^x(\text{GI})$ of gross income as follows:

$$T^{\text{fed}}(\text{GI}) = \sum_{i=1}^{I-1} \tau_i^{\text{fed}} (\bar{e}_{i+1} - \bar{e}_i) + \tau_I^{\text{fed}} (e^{\text{fed}}(\text{GI}) - \bar{e}_I),$$

$$T^{\text{state}}(\text{GI}) = \tau^{\text{state}} e^{\text{state}}(\text{GI}),$$

$$T^{\text{local}}(\text{GI}) = \tau^{\text{local}} e^{\text{local}}(\text{GI}).$$

C.2.3 Tax Credits

The OECD considers two types of federal tax credits for households without children: the Earned Income Tax Credit (EIC) and the Making Work Pay tax credit (MWP). The EIC and the MWP provide refundable tax credits equal to some fractions φ_{eic} and φ_{mwp} of gross income up to some maximum amounts $\varphi_{\text{eic}} \bar{\text{eic}}$ and $\overline{\text{mwp}}$. The tax credits are phased out at taper rates φ_{eic}^T and φ_{mwp}^T once gross income exceeds thresholds $\text{THOLD}_{\text{eic}}$ and $\text{THOLD}_{\text{mwp}}$. The total tax credit amounts from these programs are thus given by

$$\text{eic}(\text{GI}) = \max \left\{ \varphi_{\text{eic}} \min \left\{ \text{GI}, \bar{\text{eic}} \right\} - \varphi_{\text{eic}}^T \max \left\{ \text{GI} - \text{THOLD}_{\text{eic}}, 0 \right\}, 0 \right\}$$

and

$$\text{mwp}(\text{GI}) = \max \left\{ \min \left\{ \varphi_{\text{mwp}} \text{GI}, \overline{\text{mwp}} \right\} - \varphi_{\text{mwp}}^T \max \left\{ \text{GI} - \text{THOLD}_{\text{mwp}}, 0 \right\}, 0 \right\}.$$

Total federal tax credits is the sum of EIC and MWP. At the state level, the OECD includes the Michigan Earned Income Tax Credit, which is an additional refundable credit equal to a fraction φ_{meic} of the federal EIC amount. The local level incorporates the Michigan City Income Tax Credit (CTC) which is a nonrefundable credit equal to some fraction of the total local tax liability $T^{\text{local}}(\text{GI})$ up to some maximum amount $\overline{\text{ctc}}$. Below this upper bound, the CTC credit rates decline with income. Consider N credit rate brackets with marginal credit rates $\varphi_{1,\text{ctc}}, \dots, \varphi_{N,\text{ctc}}$ starting at tax liability thresholds $\bar{T}_1, \dots, \bar{T}_N$, where $\bar{T}_1 = 0$. Given a largest applicable tax credit bracket $I = \max\{i : T^{\text{local}}(\text{GI}) > \bar{T}_i\}$, the total tax credit at each level of government is then given by functions $C^x(\text{GI})$ of gross income as follows:

$$C^{\text{fed}}(\text{GI}) = \text{eic}(\text{GI}) + \text{mwp}(\text{GI}),$$

$$C^{\text{state}}(\text{GI}) = \varphi_{\text{meic}} \text{eic}(\text{GI}),$$

$$C^{\text{local}}(\text{GI}) = \min \left\{ \sum_{i=1}^{I-1} \varphi_{i,\text{ctc}} (\bar{T}_{i+1} - \bar{T}_i) + \varphi_{I,\text{ctc}} (T^{\text{local}}(\text{GI}) - \bar{T}_I), \overline{\text{ctc}} \right\}.$$

C.2.4 Effective Income Tax Rate

The effective income tax rate $\tau^w(\text{GI})$ at gross income GI is the total tax liability net of tax credits measured as a percentage of gross income:

$$\tau^w(\text{GI}) = \frac{1}{\text{GI}} \sum_{x \in X} (T^x(\text{GI}) - C^x(\text{GI})),$$

where $X = \{\text{fed}, \text{state}, \text{local}\}$. In the practical implementation of these calculations, we consider an average gross income level $\overline{\text{GI}}$ and then express all other gross incomes as a percentage of that average.

C.2.5 Estimation

Using the methodology above, I create effective income tax rates on a grid of gross incomes for each year between 2000 and 2022. The grid is linearly spaced with 401 points, ranging from 0 to a multiple 20 of average gross income. The parameter values for this exercise are collected from the OECD and are available upon request. I then fit the income tax function $\tau^w(\text{GI}) = \kappa_0 \left[1 - \left(\kappa_2 \left(\frac{\text{GI}}{\overline{\text{GI}}} \right)^{\kappa_1} + 1 \right)^{-\frac{1}{\kappa_1}} \right] + \kappa_3$ by a nonlinear OLS to these tax rates.

Figure C.2 shows the constructed tax rates for the lower end of the income grid together with the corresponding fit and its estimated coefficients. Even though this period saw two major tax reforms (the Economic Growth and Tax Reconciliation Relief Act of 2001 and the Tax Cuts and Jobs Act of 2017) and underwent three economic downturns (the early 2000s recession, the Great Recession, and the COVID-19 recession), effective income tax rates remain stable over time. Therefore, the estimated tax function provides a close fit, as seen by the high R^2 of 0.97.

C.3 Changing the Tax Rate Level While Maintaining Progressivity

Once the income tax function is estimated, I adjust its level so that the tax rate at average earnings matches the tax rate from the national accounts. To this end, I follow Guvenen, Kuruscu and Ozkan (2014) to ensure that the degree of progressivity remains the same before and after. Thus, let $\tilde{\tau}(e)$ be some tax rate function of the Gouveia and Strauss (1994) form:

$$\tilde{\tau}(e) = \tilde{\kappa}_0 \left[1 - \left(\tilde{\kappa}_2 \left(\frac{e}{\bar{e}} \right)^{\tilde{\kappa}_1} + 1 \right)^{-\frac{1}{\tilde{\kappa}_1}} \right] + \tilde{\kappa}_3. \quad (\text{C.4})$$

Denote its corresponding marginal tax rate by $\tilde{\tau}^m(e) = \frac{\partial}{\partial e} (\tilde{\tau}(e)e)$. To change the level of this tax function into a similar function $\tau(e)$ with parameters $\kappa_0, \dots, \kappa_3$ without changing the progressivity, we need the ratio of net take-home shares at any two earnings levels e and e' to be the same in both tax systems:

$$\frac{1 - \tau^m(e')}{1 - \tau^m(e)} = \frac{1 - \tilde{\tau}^m(e')}{1 - \tilde{\tau}^m(e)}.$$

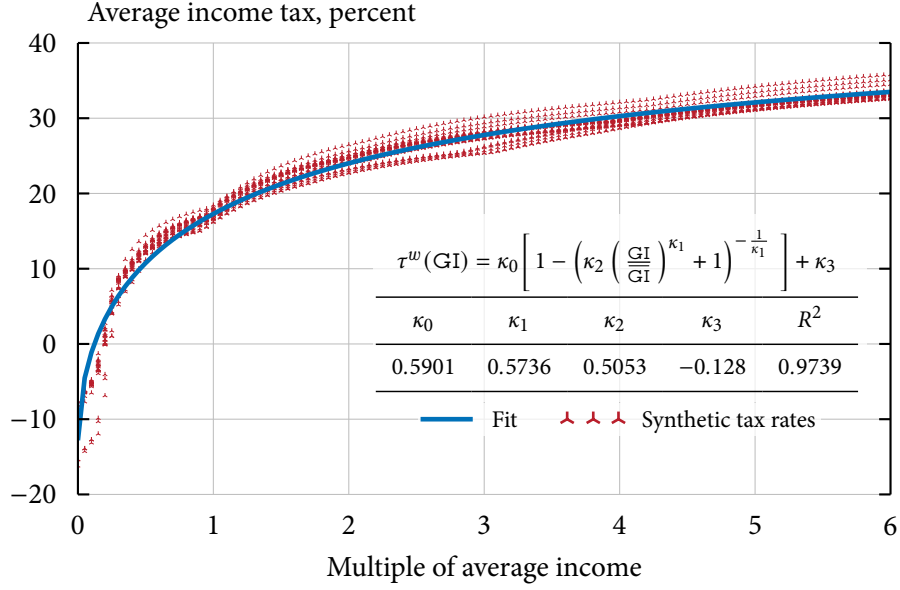


Figure C.2. Estimation of the income tax function.

This expression can be rearranged to obtain

$$\tau^m(e) = 1 - \bar{k}(1 - \tilde{\tau}^m(e)), \quad \text{where} \quad \bar{k} \equiv \frac{1 - \tau^m(e')}{1 - \tilde{\tau}^m(e')} \quad (\text{C.5})$$

is a level ratio between the two tax systems that we are free to choose. Since $\tau(e)e = \int_0^e \tau^m(x) dx$, we can integrate Equation (C.5) to obtain an average tax rate of a similar form:

$$\tau(e) = 1 - \bar{k}(1 - \tilde{\tau}(e)). \quad (\text{C.6})$$

Substituting Equation (C.4) into (C.6) and rearranging terms, we finally get

$$\tau(e) = \kappa_0 \left[1 - \left(\kappa_2 \left(\frac{e}{\bar{e}} \right)^{\kappa_1} + 1 \right)^{-\frac{1}{\kappa_1}} \right] + \kappa_3,$$

where $\kappa_0 \equiv \bar{k} \cdot \tilde{\kappa}_0$, $\kappa_1 \equiv \tilde{\kappa}_1$, $\kappa_2 \equiv \tilde{\kappa}_2$ and $\kappa_3 \equiv 1 - \bar{k}(1 - \tilde{\kappa}_3)$. The calibrated $\kappa_0, \dots, \kappa_3$ in the benchmark model use the estimates in Figure C.2 as $\tilde{\kappa}_0, \dots, \tilde{\kappa}_3$ and set the scale parameter $\bar{k}$ such that the tax rate at average income $\bar{e}$ matches the income tax rate τ^{NA} from the national accounts. Specifically, the latter requires that $\tau(\bar{e}) = \tau^{NA} = 1 - \bar{k}(1 - \tilde{\tau}(\bar{e}))$, which can be rearranged to give the scale parameter as

$$\bar{k} = \frac{1 - \tau^{NA}}{1 - \tilde{\tau}(\bar{e})}, \quad \text{where} \quad \tilde{\tau}(\bar{e}) = \tilde{\kappa}_0 \left[1 - \left(\tilde{\kappa}_2 + 1 \right)^{-\frac{1}{\tilde{\kappa}_1}} \right] + \tilde{\kappa}_3$$

is a function of estimated parameters only.

Appendix D Internal Calibration of Model Parameters

The internally calibrated parameters are obtained by a method of moments procedure using model-predicted and empirical data. Denote the vector of parameters by $\Theta = (\beta, \psi, \Omega_A, \delta_k, \delta_z, \alpha, \rho)$. Let y_t be

the vector of empirical observations in period t for the variables whose moments we wish to match, and let $f_t(\Theta)$ be the corresponding model-predicted observations as a function of the parameters. The residual between model and data is then $u_t(\Theta) = y_t - f_t(\Theta)$ for $t = 1, \dots, T$. I obtain the parameter vector from the method of moments estimator by setting the sample averages of $u_t(\Theta)$ to zero (up to some tolerance):

$$\hat{\Theta} = \arg \min_{\Theta} \left(\frac{1}{T} \sum_{t=1}^T u_t(\Theta) \right)^{\top} \left(\frac{1}{T} \sum_{t=1}^T u_t(\Theta) \right).$$

The moment estimators for α and ρ have analytical solutions and can be directly determined from the data. The remaining parameters are estimated by simulation. The practical implementation of this estimation is the same as in Ludwig, Schelkle and Vogel (2012), which I refer to for further details. In short, the algorithm proceeds as follows:

- (i) Start with the empirical moments as calibration targets.
- (ii) Use these targets to calibrate the model out of steady state.
- (iii) For the estimated parameters, compute the equilibrium transition path.
- (iv) Obtain the corresponding model-predicted moments for the relevant transition periods.
- (v) If the predicted moments exceed the empirical moments, decrease the steady-state calibration target (and vice versa).
- (vi) Iterate until the predicted moments converge to the empirical counterparts.

Because the model begins before most official US data series, constructing the empirical moments requires some special care. The following summarises the data choices for the internal calibration.

- **Output:** I use Kendrick's (1961, Table A-III) real GDP series until 1929, inflate it with the implicit GNP deflator from his Tables A-IIa and A-IIb, and splice it with the BEA's GDP series (NIPA Table 1.1.5).
- **Capital stock:** I sum structures, producer durables, and nonfarm land from Goldsmith, Brady and Mendershausen's (1956) Table W-1 and splice it with BEA data on total fixed assets net of IPP assets (FAA Table 1.1).
- **Hours worked:** I splice Kendrick's (1961, Table A-X) aggregate hours estimates with Bureau of Labor Statistics data on hours for all workers in the total US economy. I then divide by the population aged 20 to 64, as reported in the Census Bureau's intercensal population tables, and convert to the unit interval assuming a 16-hour daily time endowment. The estimation considers normalized hours worked, $h/Z^{\theta(1-\sigma)/(1+\theta\sigma)}$, where Harrod-neutral TFP is obtained as the residual between gross output and labour and capital inputs in Kendrick's Table A-XIX using the labour shares below.
- **Life-cycle profile of assets in 1950:** I take this from the 1949 Survey of Consumer Finances, as reported in Goldsmith (1955, Table XLIII).
- **Depreciation of non-IPP capital:** before 1929, I use Goldsmith, Brady and Mendershausen's (1956, Table N-5) estimates and splice them with BEA data on total consumption of fixed capital less intellectual property products (NIPA Table 5.2.5).
- **Depreciation of IPP capital:** this does not exist before the NIPA start year, which restricts this moment to the 1929–1949 subperiod when it is available (NIPA Table 5.2.5).
- **Labour share:** I extend the BEA's national income data (NIPA Table 1.12) back to 1900 using income components from Martin (1939, Tables 4 and 41) and Goldsmith, Brady and Mendershausen (1956, Tables N-1 and N-5). The labour share is calculated as compensation of employees divided by GDP net of subsidies, taxes on production and imports, and proprietors' income. The latter subtraction

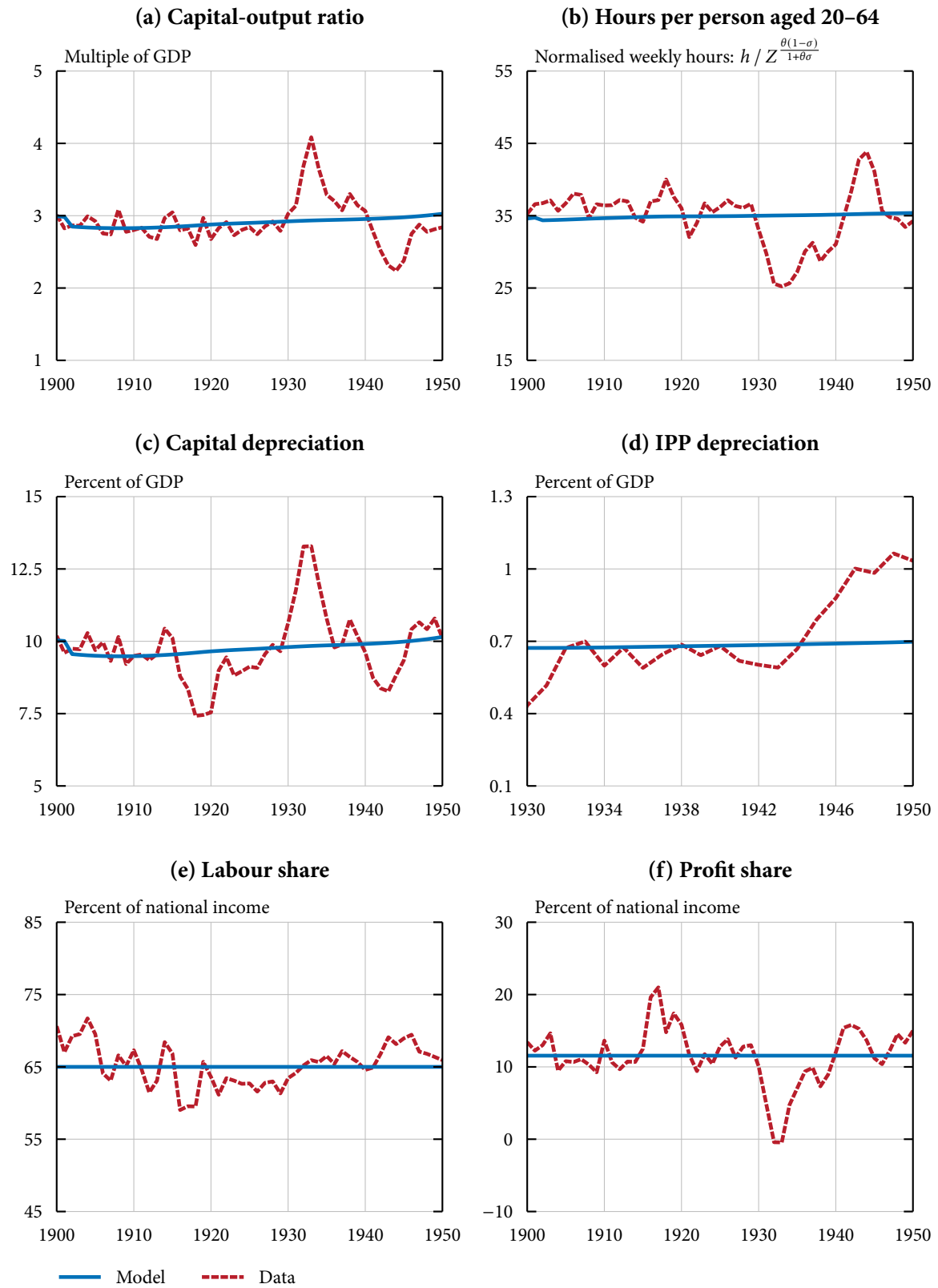


Figure D.1. Time series for simulated method of moments.

imposes the assumption that the self-employed use labour as intensively as the rest of the economy, per recommendation of Karabarbounis (2024).

- **Profit share:** is constructed analogously to the labour share using corporate profits—the sum of taxes on corporate income, net dividends, and undistributed corporate profits—as numerator, again assuming the same profit share for proprietors as in the corporate sector.

Figure D.1 plots the resulting empirical time series against the model's predictions (the life-cycle profile of assets is already shown in Figure 4a in the main body of the paper). The figure reveals that the model closely fits the data on average, with most deviations being due to the large swings associated with the Great Depression and its subsequent recovery.

Appendix E Additional Sensitivity Analyses

This appendix complements Section 7 in the main paper with additional robustness checks. Each robustness scenario recalibrates the parameters β , ψ , Ω_a , δ_k , and δ_z to match the baseline calibration targets if needed. The growth rate of each scenario is plotted against the baseline in Figure E.1, while Figure E.2 shows their growth decompositions. Overall, neither alternative alters any qualitative conclusions.

Increased retirement age. The social security system in the benchmark model has a fixed normal retirement age (NRA) and early/delayed pension scaling schedule, contrary to reality. Since the average age of retirement influences all growth mechanisms in the paper, I therefore consider an alternative which more accurately describes the NRA and the delayed retirement credits. Specifically, I increase the NRA to 66 for cohorts born between 1940 and 1956 and to 67 for subsequent cohorts. Moreover, the delayed retirement credit is increased by 0.5 percentage points for every other cohort between 1924 and 1943. That is, the delayed retirement credit is 3 percent for cohorts born before 1925, 3.5 percent for the 1925–1926 cohorts, . . . , 7.5 percent for the 1941–1942 cohorts, and 8 percent for all subsequent cohorts. Overall, these changes nevertheless leave the baseline results unaffected because most households in the baseline already retire between the ages of 65 and 70.

Deterministic earnings. In the baseline, households face uninsurable idiosyncratic productivity shocks that add a savings motive beyond the standard life-cycle motive. In an alternative, I remove this feature and consider deterministic earnings. Although this reduces inequality and the overall level of household savings, the difference to the baseline turns out to be negligible. The large one-off growth spike in this simulation is not a numerical error. Rather, this is a year in which households choose to start retiring one year later than prior cohorts. Since the model is deterministic, this change applies to everyone within a cohort, thus generating a large increase in the employment rate in this particular period.

No bequest motive. Without a bequest motive, an increase in life expectancy strengthens the life-cycle savings motive and unambiguously raises the saving rate. Warm-glow bequests act as a counteracting force, since higher survival rates dampen the savings motive of leaving bequests. To assess whether this is an important aspect, I set the utility weight of bequests (Ω_a) to zero and rerun the model. As expected, the growth impact is now larger, although it is quantitatively insignificant and changes no insights from the baseline.

Flat income tax. Rather than considering a progressive income tax, I analyse an alternative in which all households face a constant marginal (and average) income tax rate equal to that obtained from the national accounts: $\tau^w = 0.116$. Contrary to the deterministic earnings scenario, imposing a flat tax

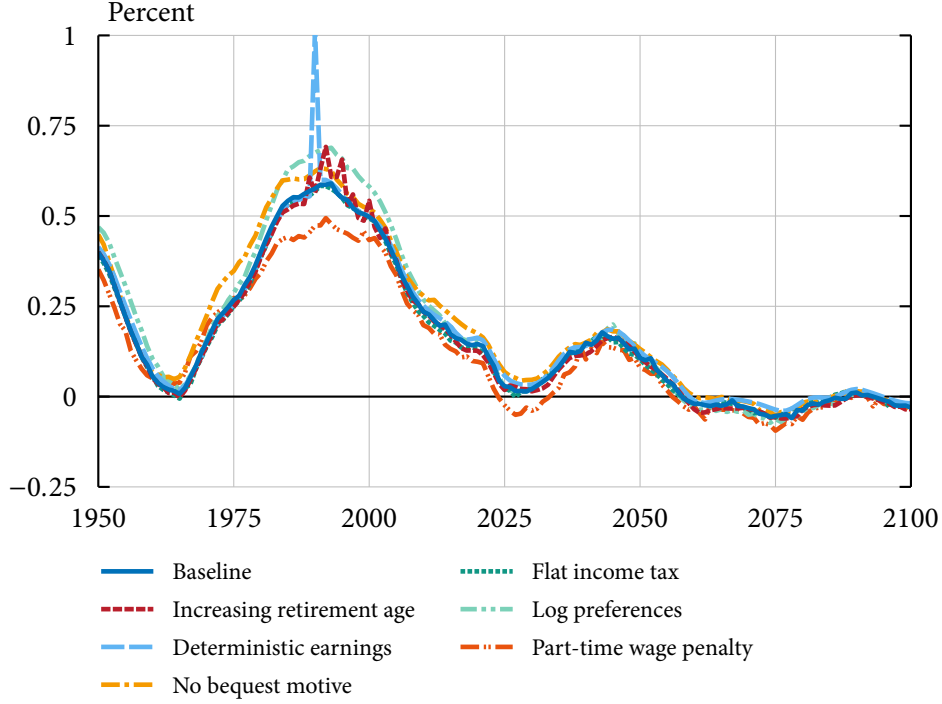


Figure E.1. Sensitivity check: growth rates with different model specifications.

increases inequality and the level of household savings. Again, the difference to the baseline is nevertheless negligible.

Log preferences. Another nonstandard feature is that I consider preferences of the Boppart and Krusell (2020) class that generate declining hours worked along a balanced growth path. By contrast, a large part of the macroeconomic literature restricts itself to the subset of this class defined by King, Plosser and Rebelo (1988), in which hours worked are constant in the long run. Constant long-run hours in my model are obtained as the special case when $\sigma \rightarrow 1$, so that flow utility becomes $u(c_j, h_j, a_{j+1}^b) = \Omega_{jc} \log(c_j) - \psi \frac{h_j^{1+1/\theta}}{1+1/\theta} + (1-s_j)\Omega_a \log(a_{j+1}^b)$. In this case, the income and substitution effect on leisure exactly offset each other, so hours worked do not fall when growth is positive. Between 1950 and 2000, annual growth with logarithmic preferences is therefore about 0.1 percentage points higher. This difference is explained entirely by the different adjustments in hours worked. Growth during the twenty-first century does not change since, with wage growth around zero, the response in hours is similar in both scenarios.

Part-time wage penalty. Several authors stress the importance of nonconvexities in the budget set to generate endogenous retirement (see for instance Rogerson and Wallenius, 2013, and Ljungqvist and Sargent, 2014). Social security plays this role in the baseline model. Another commonly used nonconvexity is nonlinear wages, which is motivated by the empirical observation that part-time work does not pay as high hourly wage as full-time work. Thus, following French (2005), consider household labour earnings given by $w\epsilon_j\eta h_j^{1+\xi}$, with $\xi \geq 0$. The labour market condition then changes to $L = \sum_j \int_X \epsilon_j \eta h_j(x)^{1+\xi} d\Phi_j$ and, by similar derivations as in Appendix A, the long-run growth rate of intermediate firms becomes

$$1 + g_z = (1 + n)^\chi, \quad \text{where} \quad \chi \equiv \frac{\lambda}{1 - \phi - \lambda \frac{1+\theta}{1+\theta\sigma - \xi\theta(1-\sigma)} \frac{\alpha}{1-\alpha} \frac{1-\rho}{\rho}}.$$

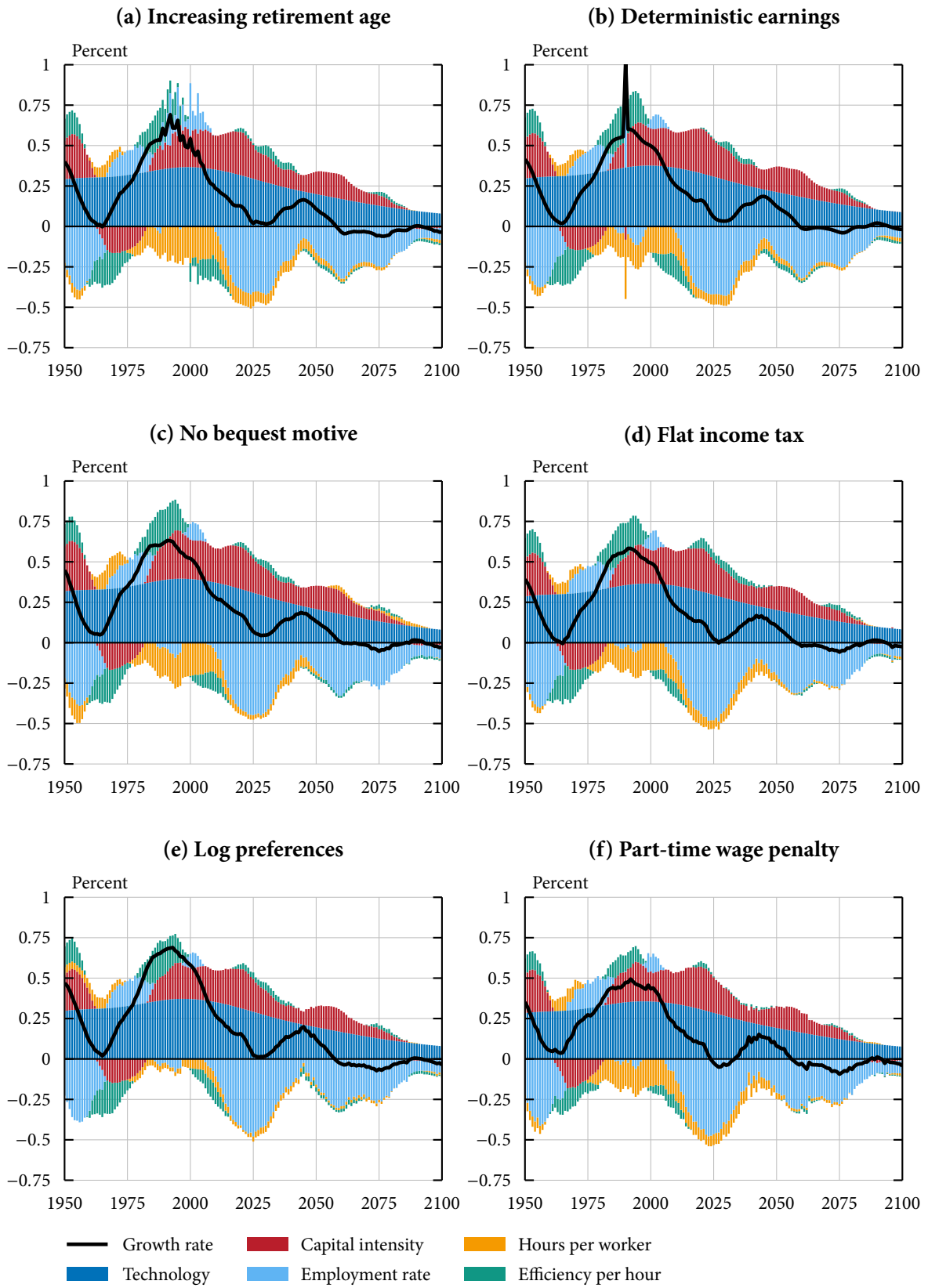


Figure E.2. Sensitivity check: growth decompositions with different model specifications.

The long-run growth rates of output per capita and hours per worker similarly become

$$1 + g_y = (1 + g_Z)^{\frac{1+\theta}{1+\theta\sigma-\xi\theta(1-\sigma)}} \quad \text{and} \quad 1 + g_h = (1 + g_Z)^{\frac{\theta(1-\sigma)}{1+\theta\sigma-\xi\theta(1-\sigma)}}.$$

For $\xi = 0$, wages are a linear function of hours worked and we obtain the benchmark model. Here, I follow French and set the value of ξ to 0.415 based on Aaronson and French's (2004) empirical finding that a 50 percent reduction in hours corresponds to a 25 percent lower hourly wage. I recalibrate the model under the assumption that $\xi = 0.415$ holds, which lowers the intertemporal elasticity of substitution, $\sigma = 1.84$, and flattens the age-efficiency profile.⁴

With these adjustments, the growth rate is similar to the baseline on average but exhibits more stable dynamics during the twentieth century. The latter is due to two changes to the average productivity per hour worked. First, declining hours worked negatively impacts average productivity since the productivity of an individual worker, $\varepsilon_j \eta h_j^\xi$, now includes hours worked. Second, the age-efficiency profile is flatter than in the baseline. Changes in the age composition of the labour force, which are more prominent in the twentieth century, therefore leads to smaller changes in average efficiency.

⁴ For the baseline, I estimate the age-efficiency profile $\{\varepsilon_j\}_{j=1}^J$ from a PSID wage measure obtained by dividing annual labour income with annual hours. Here, I assume that $\xi = 0.415$ holds and construct PSID wages as (total annual labour income)/(annual hours worked)^{1.415} (see Appendix B for estimation details).

Appendix F Additional Figures

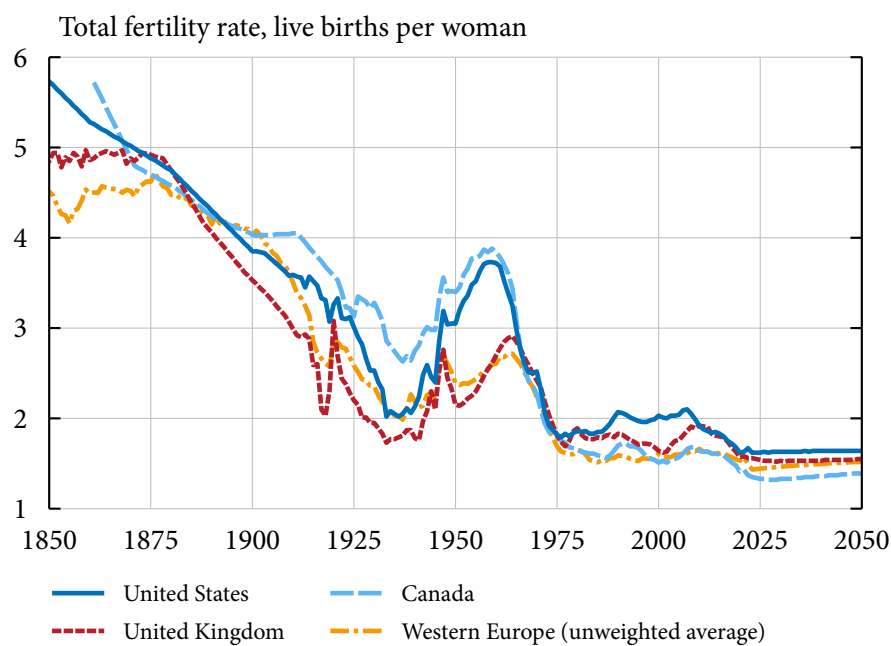


Figure F.1. Fertility across major economies.

Source. Gapminder Foundation (2024).

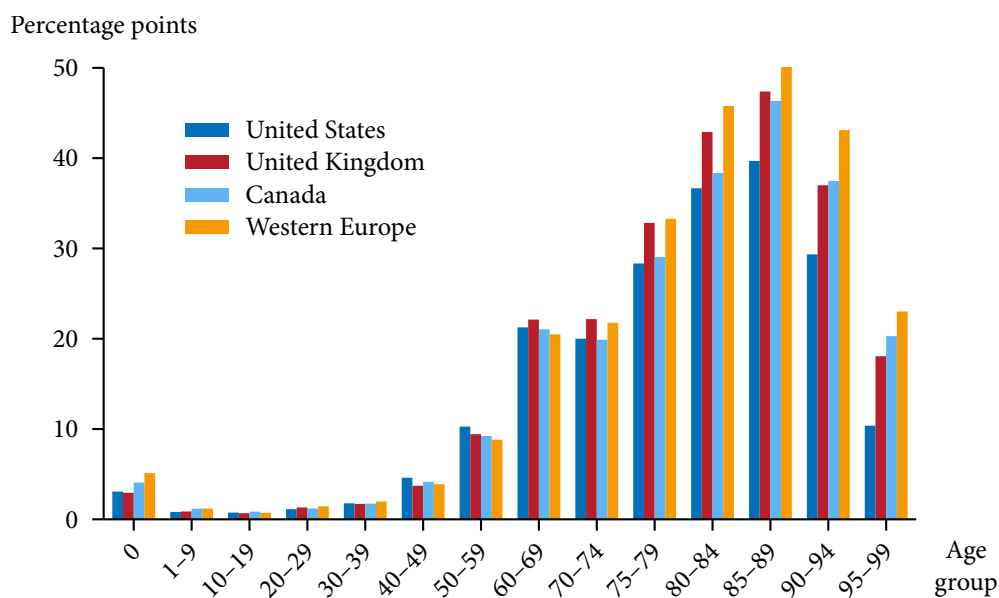


Figure F.2. Increase in survival probabilities by age group across major economies, 1950–2100.

Source. United Nations (2024).

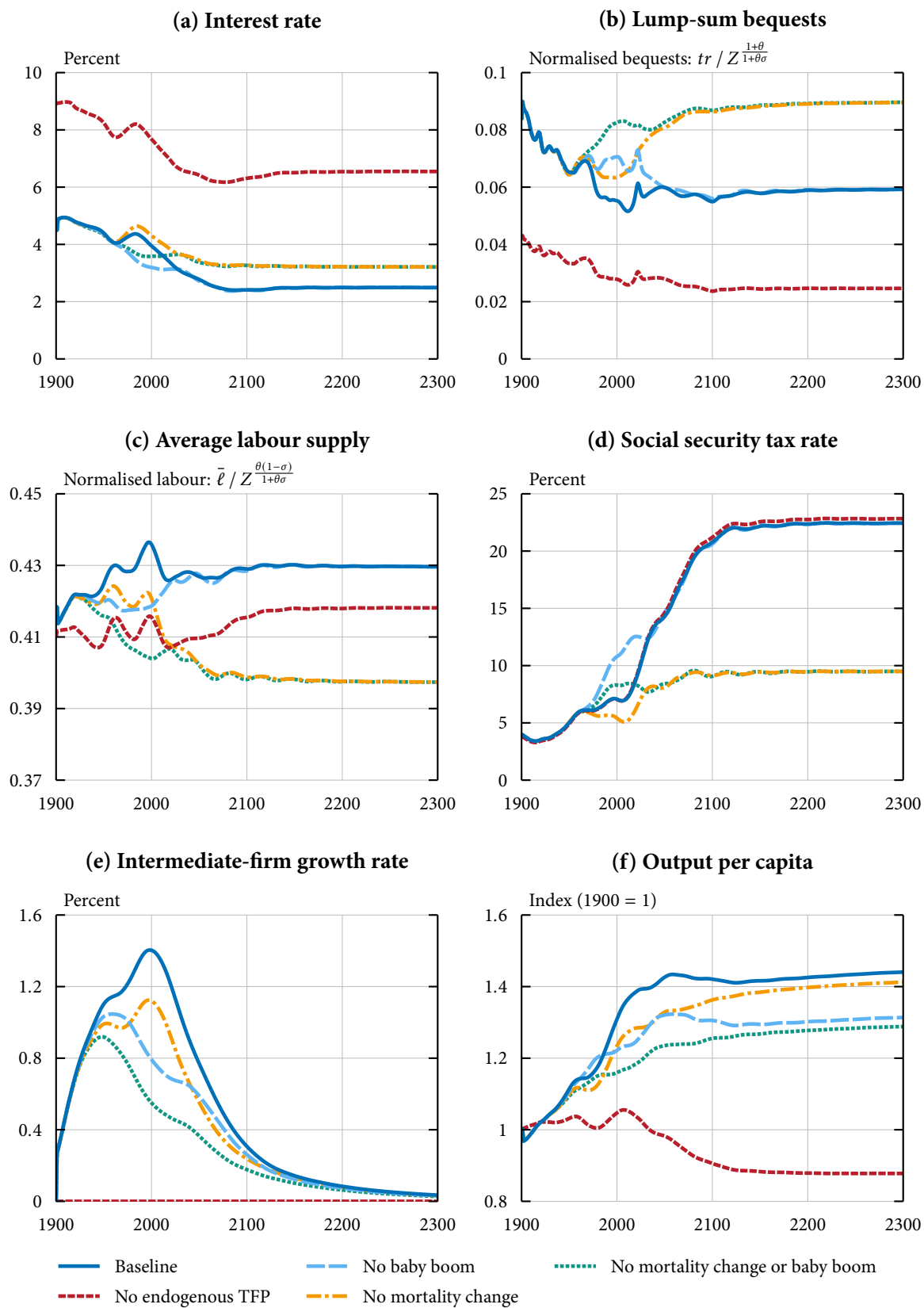


Figure F.3. Transition paths of equilibrium variables and output per capita.

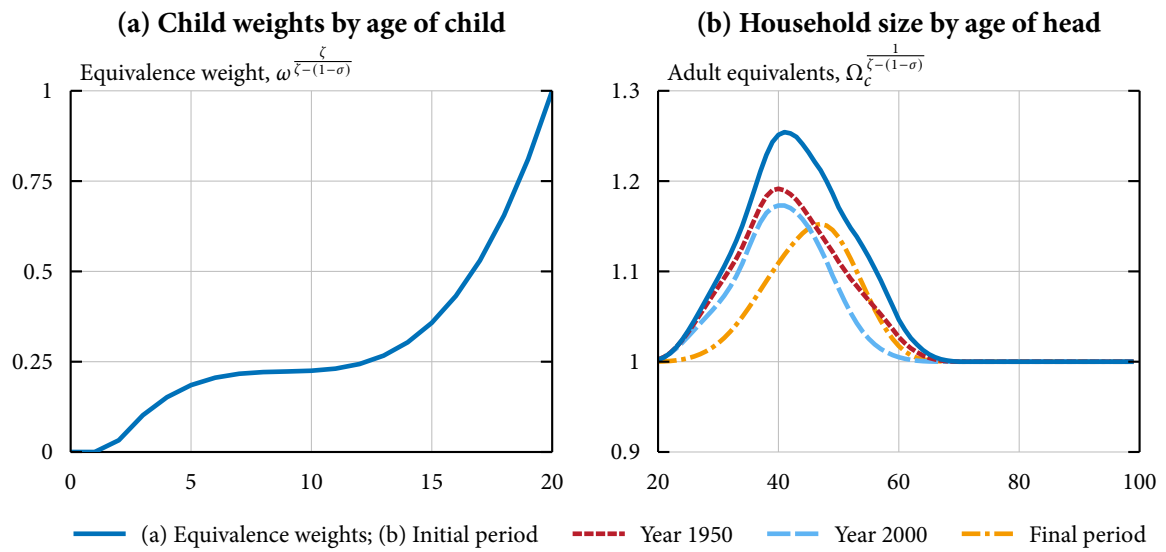


Figure F.4. Household size.

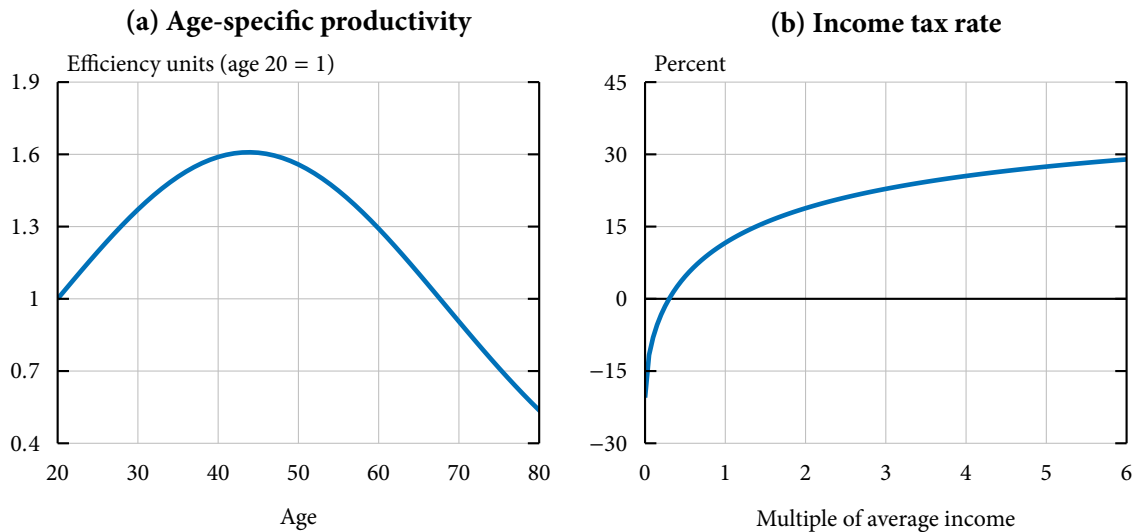


Figure F.5. Calibrated productivity and tax profiles.

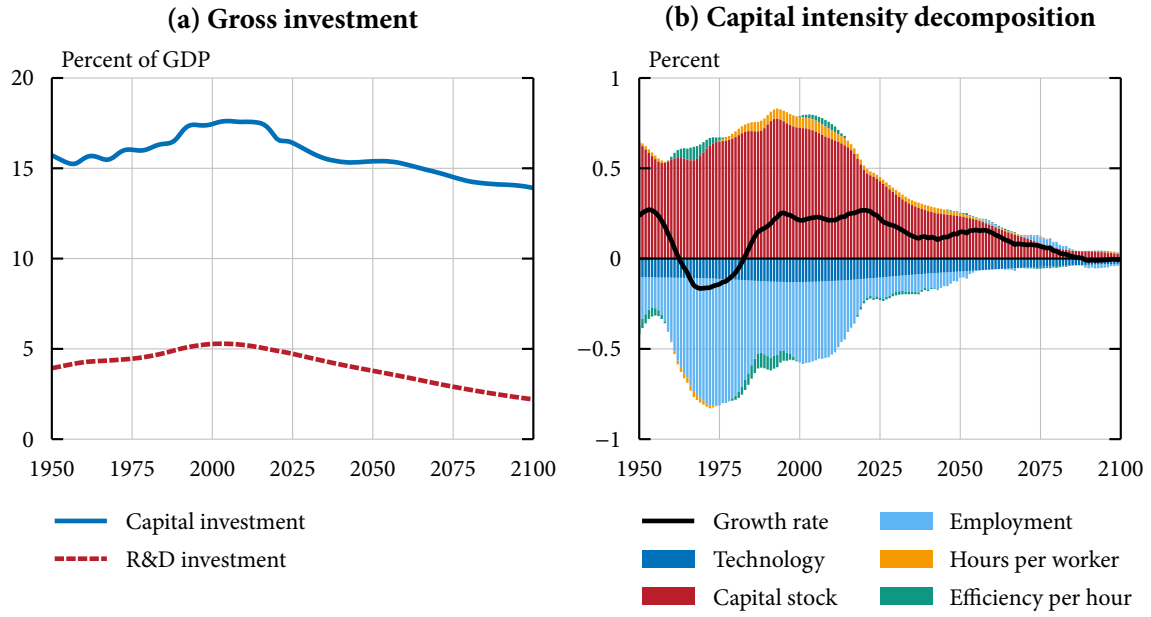


Figure F.6. Investment and capital deepening in the baseline scenario.

Notes. Figure F.6b displays the growth decomposition $\frac{\alpha}{1-\alpha} g_{K/Y} = \alpha(g_K - g_Z - g_E - g_h - g_\epsilon)$, which is obtained by log differencing the capital intensity $\left(\frac{K}{Y}\right)^{\frac{\alpha}{1-\alpha}} = \left(\frac{K}{ZL}\right)^\alpha$ together with the labour decomposition $L = E \bar{h} \bar{\epsilon}$.

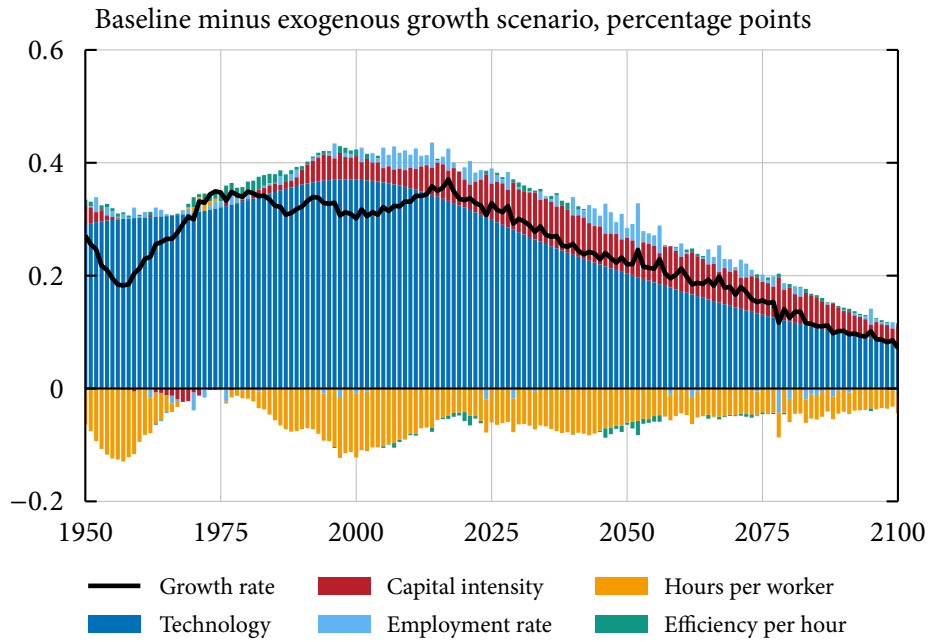


Figure F.7. Growth difference between the baseline and exogenous growth scenario.

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