Online Appendix (Not for publication)

A Numerical Example

We provide a numerical example for the crowding-out effect, illustrating the existence of relevant parameters in Propositions 3-7. This example highlights the qualitative (rather than quantitative) aspect of the model. The numerical examples for Propositions 8 and 10 will be provided in their proofs.

We choose parameter values as simple as possible. We set the parameter values for the project as $I = 1$, $C = 2.2$, $\pi = 0.4$, $\mathbb{E}^H(\widetilde{x}) = 1.3668$, $\mathbb{E}^L(\widetilde{x}) = 1.0334$. Note that we do not need to specify exactly u, d, θ_H and θ_L, any combination of which that satisfies that $u > d > 0$, $0 \leq X_1 < X_2 \leq d$, $u \cdot \theta_H + d \cdot (1 - \theta_H) = 1.3668$ and $u \cdot \theta_L + d \cdot (1 - \theta_L) = 1.0334$ works. The distribution of B is $f(B) = \log \frac{1}{1-B}$ for $B \in [0, 1)$. As the maximum total amount of liquidity that any one sector can demand is $Q^{\max} = \int_0^1 Bf(B)dB = 0.75$, we set $\overline{Q} = 2Q^{\max} = 1.5$.

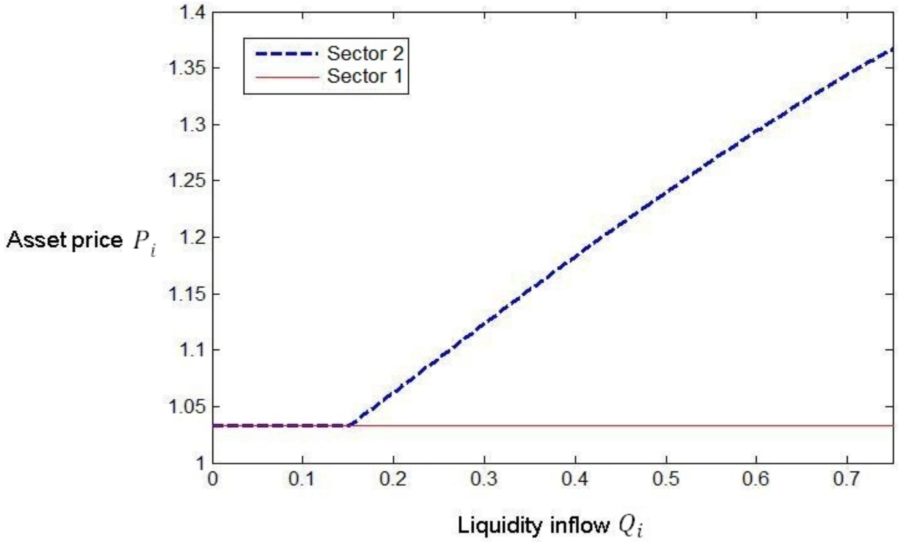
Given that $\mathbb{E}^L(\widetilde{x}) = 1.0334$, we can calculate the threshold $\underline{X}$ in Proposition 3, which is $\underline{X} = 0.0501$. We set $X_1 = 0.05$ and $X_2 = 0.35$, where $X_1 < \underline{X} < X_2$.

For Sector 1, we can work out that the asset price is $P_1 = 1.0334$ for any $Q_1 \in [0, 0.75]$. The asset price is trapped at $\mathbb{E}^L(\widetilde{x}) = 1.0334$. The interest rate $r_1(Q_1)$ is a strictly decreasing function of liquidity inflow Q_1.

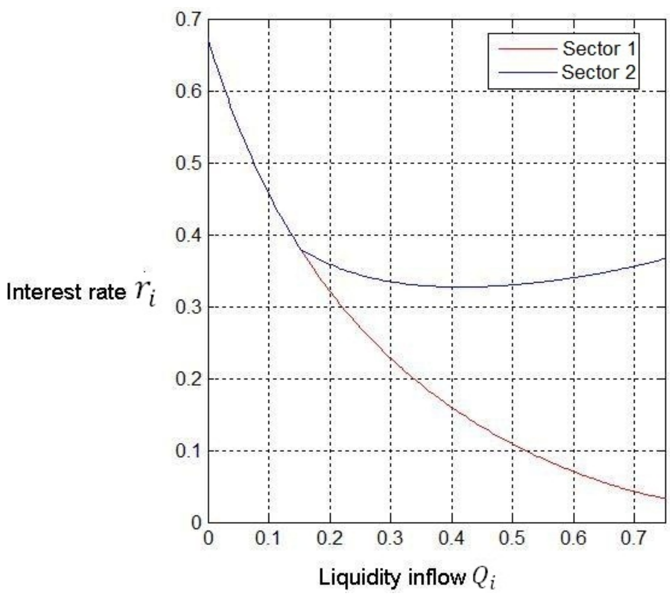
For Sector 2, the asset price $P_2(Q_2)$ is (weakly) increasing in Q_2. When Q_2 is small, liquidity injections are not sufficient to push the asset price above $\mathbb{E}^L(\widetilde{x})$ and the asset price is $P_2 = 1.0334$; when Q_2 is big enough, $P_2(Q_2)$ is strictly increasing in Q_2, with the maximum asset price being $P_2(Q_2 = 0.75) = 1.3667$ (which is lower than $\mathbb{E}^H(\widetilde{x}) = 1.3668$). As for the equilibrium interest rate $r_2(Q_2)$, it is non-monotonic and 'U'-shaped, with the minimum interest rate being $\mathrm{r}_{\min} = 0.3270$.

The optimal amount of liquidity injection to maximize liquidity investments in Sector 1 is $Q^* = 0.6113$, at which the distribution of the liquidity injection across the two sectors is $Q_1^* = 0.1950$ and $Q_2^* = 0.4163$, respectively.

Figure A1(a) shows the asset price response in each sector to its (net) liquidity inflow Q_i and Figure A1(b) depicts the interest rate response in each sector.



(a) Asset price response to liquidity inflow



(b) Interest rate response to liquidity inflow

Figure A1: A numerical example

B Proofs

Proof of Lemma 2: By using the credit market clearing condition (2), the expression of $\Gamma(B^*, r)$ in (3) can be rewritten as

$$\Gamma\left(B^*, r\right) = \frac{\pi\left[C \cdot F\left(B^*\right) - Q\left(1+r\right)\right] + X}{1-\pi}.$$

By substituting (1) into this equation, r can be eliminated. That is,

$$\Gamma\left(B^*\right) = \frac{\pi C \cdot F\left(B^*\right) + X}{\left(1-\pi\right) + \pi \frac{Q}{B^*}}.$$

From (2), B^* is completely and uniquely determined by Q. Hence, Γ can be expressed in terms of Q:

$$\Gamma\left(Q\right) = \frac{\pi C \cdot F\left(B^*\left(Q\right)\right) + X}{\left(1-\pi\right) + \pi \frac{Q}{B^*(Q)}}.$$

Now we derive the total derivative $\frac{d\Gamma}{dQ}$.

From (2), we can work out
$$\frac{dB^*}{dQ} = \frac{1}{If\left(B^*\right)}. \tag{A1}$$

From the expression $\Gamma(B^*, r)$ in (3), we have the total derivative:

$$\begin{aligned}
\frac{d\Gamma}{dB^*} &= \frac{\partial \Gamma}{\partial B^*} + \frac{dr}{dB^*}\frac{\partial \Gamma}{\partial r} \\
&= \frac{\partial \Gamma}{\partial B^*} + \frac{d}{dB^*}\left(\frac{P}{B^*} - 1\right)\frac{\partial \Gamma}{\partial r} \\
&= \frac{\partial \Gamma}{\partial B^*} + \left(\frac{1}{B^*}\frac{dP}{dB^*} - \frac{P}{B^{*2}}\right)\frac{\partial \Gamma}{\partial r},
\end{aligned} \tag{A2}$$

where the equality in the second line uses the optimal lending condition (1). Given the interior region of (3) in which $P = \Gamma$, we can rewrite equation (A2) as

$$\frac{d\Gamma}{dB^*} = \frac{\partial \Gamma}{\partial B^*} + \left(\frac{1}{B^*}\frac{d\Gamma}{dB^*} - \frac{\Gamma}{B^{*2}}\right)\frac{\partial \Gamma}{\partial r}. \tag{A3}$$

Noticing that the term $\frac{d\Gamma}{dB^*}$ appears on both sides of equation (A3), we solve for this term

$$\frac{d\Gamma}{dB^*} = \frac{\frac{\partial \Gamma}{\partial B^*} - \frac{\Gamma}{B^{*2}}\frac{\partial \Gamma}{\partial r}}{1 - \frac{1}{B^*}\frac{\partial \Gamma}{\partial r}}. \tag{A4}$$

Also, from the expression of $\Gamma(B^*, r)$ in (3), we have

$$\begin{aligned}
\frac{\partial \Gamma}{\partial B^*} &= \frac{\pi}{1-\pi}\left[C - I\left(1+r\right)\right] f\left(B^*\right), \\
\frac{\partial \Gamma}{\partial r} &= \frac{\pi\left\{\int_0^{B^*} -Bf\left(B\right)dB + \int_{B^*}^{I}\left(I-B\right)f\left(B\right)dB\right\}}{1-\pi} = -\frac{\pi}{1-\pi}Q,
\end{aligned}$$

where the second equality follows from the credit market clearing condition (2).

Therefore, we can use these expressions to simplify expression (A4):

$$\frac{d\Gamma}{dB^*} = \frac{\left[C - I\left(1+r\right)\right] f\left(B^*\right) + \left(1+r\right)\frac{Q}{B^*}}{\frac{1-\pi}{\pi} + \frac{Q}{B^*}}. \tag{A5}$$

Overall, we have

$$\frac{d\Gamma}{dQ} = \frac{dB^*}{dQ}\frac{d\Gamma}{dB^*} = \frac{\left[C - I\left(1+r\right)\right] f\left(B^*\right) + \left(1+r\right)\frac{Q}{B^*}}{If\left(B^*\right)\left(\frac{1-\pi}{\pi} + \frac{Q}{B^*}\right)}. \tag{A6}$$

By (A6), we conclude that a sufficient condition for $\frac{d\Gamma}{dQ} > 0$ is $C - I(1+r) > 0$.

The intuition behind Lemma 2 is the following. From (3), $\frac{d\Gamma}{dB^*} = \frac{\pi}{1-\pi}\{\left[C - I\left(1+r\right)\right] f\left(B^*\right) - \frac{dr}{dB^*}Q\}$. Liquidity injections Q enable more entrepreneurs, who are otherwise unable, to make the liquidity investment. Suppose that in the economy there is one more entrepreneur switching from non-investing to investing at T_0. Given r, this would increase the liquidity in the industry at T_1 by an amount $C - I\left(1+r\right)$, which corresponds to the NPV of the marginal investment in (4). Further, in the general equilibrium, r changes, which has the effect of the term $\frac{dr}{dB^*}Q$. However, in the general equilibrium, the first term always dominates the second term (i.e., $\left[C - I\left(1+r\right)\right] f\left(B^*\right) - \frac{dr}{dB^*}Q > 0$) when $C - I\left(1+r\right) > 0$. In fact, if $\frac{dr}{dB^*}$ is positive, we immediately conclude that Γ must increase in Q because $\Gamma = B^*\left(1+r\right)$ by (1) and B^* is increasing in Q. If $\frac{dr}{dB^*}$ is negative, clearly $\left[C - I\left(1+r\right)\right] f\left(B^*\right) - \frac{dr}{dB^*}Q > 0$, meaning $\frac{d\Gamma}{dQ} > 0$. In short, if $C - I(1+r)$ is positive and close to zero, $\frac{dr}{dB^*}$ must be negative; if $\frac{dr}{dB^*}$ is positive, $C - I\left(1+r\right)$ must be far above zero and its effect must exceed the effect of $\frac{dr}{dB^*}$.

Proof of Proposition 3: For ease of exposition, we reiterate the equilibrium asset price here. That is

$$P = \begin{cases} \mathbb{E}^H\left(\widetilde{x}\right) & \text{if} \quad \Gamma\left(B^*, r\right) > \mathbb{E}^H\left(\widetilde{x}\right) \\ \Gamma\left(B^*, r\right) & \text{if} \quad \Gamma\left(B^*, r\right) \in \left[\mathbb{E}^L\left(\widetilde{x}\right), \mathbb{E}^H\left(\widetilde{x}\right)\right], \\ \mathbb{E}^L\left(\widetilde{x}\right) & \text{if} \quad \Gamma\left(B^*, r\right) < \mathbb{E}^L\left(\widetilde{x}\right) \end{cases}$$

where

$$\Gamma\left(B^{*},r\right)=\frac{\pi\left[\int_{0}^{B^{*}}\left[C-B\left(1+r\right)\right]f\left(B\right)dB+\int_{B^{*}}^{I}\left(1+r\right)\left(I-B\right)f\left(B\right)dB\right]+X}{1-\pi}.$$

From Lemma 2, we know that $\Gamma(Q)$ is an increasing function of Q if $C-I(1+r)>0$. We show that the condition $C-I(1+r)>0$ holds under a broad range of parameter values. In fact, by $1+r=\frac{\Gamma(B^{*},r)}{B^{*}}$, we have $C-I(1+r)=C-I\frac{\Gamma(B^{*},r)}{B^{*}}$, where B^{*} is completely determined by Q and r is endogenous. *Ceteris paribus*, if π is sufficiently low, Γ is low and thus $C-I(1+r)>0$. The numerical example in Appendix A is one case of this.

We consider the lower bound of the asset price, $\mathbb{E}^{L}(\widetilde{x})$. For our purpose, we choose the parameters to ensure that the upper bound of the asset price is not binding (i.e., the asset price calculated in $\Gamma(Q)$ is always below $\mathbb{E}^{H}(\widetilde{x})$). We define two cutoffs, $\underline{X}$ and $\overline{X}$, and divide X into three ranges: $X<\underline{X}$, $X>\overline{X}$, and $\underline{X}\leq X\leq\overline{X}$.

The first range of X is the case in which the asset price is trapped at the lower bound no matter what Q ($\in[0,\overline{Q}]$) is. That is, even if $Q=\overline{Q}$, the asset price calculated in Γ is still (weakly) below $\mathbb{E}^{L}(\widetilde{x})$. Therefore, $\underline{X}$ satisfies $\mathbb{E}^{L}(\widetilde{x})=\Gamma|_{Q=\overline{Q},\ X=\underline{X}}$.

The third range of X is the case in which the asset price is above the lower bound for the whole region of $Q\in[0,\overline{Q}]$. That is, even if $Q=0$, the asset price calculated in Γ is still (weakly) above $\mathbb{E}^{L}(\widetilde{x})$. Therefore, $\overline{X}$ satisfies $\mathbb{E}^{L}(\widetilde{x})=\Gamma|_{Q=0,\ X=\overline{X}}$.

In the second range of $\underline{X}\leq X\leq\overline{X}$, the asset price is the constant $\mathbb{E}^{L}(\widetilde{x})$ when Q is low and then increases in Q when Q is higher.

Proof of Proposition 4: When $X\leq\underline{X}$, the asset price P is constant in Q. Also considering $\frac{dB^{*}}{dQ}>0$, we have that when $X\leq\underline{X}$, the equilibrium interest rate $r(Q)$ is strictly decreasing in Q ($\in[0,\overline{Q}]$).

We consider the equilibrium interest rate when $X>\underline{X}$. By $r=\frac{\Gamma}{B^{*}}-1$, we have

$$\frac{dr}{dB^{*}}=\frac{1}{B^{*}}\frac{d\Gamma}{dB^{*}}-\frac{\Gamma}{B^{*2}}.\tag{A7}$$

Plugging (A5) into (A7), we have

$$\frac{dr}{dB^{*}}=\frac{\left[C-I\left(1+r\right)\right]f\left(B^{*}\right)B^{*}-\frac{1-\pi}{\pi}\Gamma}{\left(\frac{1-\pi}{\pi}B^{*}+Q\right)B^{*}}.\tag{A8}$$

By $1+r=\frac{P(Q)}{B^*(Q)}$ and considering that $P=\Gamma$, we have

$$\begin{aligned}\frac{dr}{dQ} &= \frac{dB^*}{dQ}\frac{dr}{dB^*} \\ &= \frac{1}{If(B^*)}\frac{[C-I(1+r)]f(B^*)B^*-\frac{1-\pi}{\pi}\Gamma}{\left(\frac{1-\pi}{\pi}B^*+Q\right)B^*} \\ &= \frac{[C-I(1+r)]B^*-\frac{1-\pi}{\pi}\frac{\Gamma}{f(B^*)}}{\left(\frac{1-\pi}{\pi}B^*+Q\right)B^*I}. \end{aligned} \tag{A9}$$

Hence, $\frac{dr}{dQ}>0$ if and only if $f(B^*)>\frac{1-\pi}{\pi}\left[\frac{C}{1+r}-I\right]^{-1}$ by noting that $\frac{\Gamma}{B^*}=1+r$. In equilibrium, we guarantee that $C>I(1+r)$, so the right-hand side of this condition is bounded. Note that by the proof of Lemma 2, if $\frac{dr}{dB^*}$ is positive, $C-I(1+r)$ must be greater than a positive number. When $f(B^*)$ is sufficiently high, $\frac{dr}{dQ}$ is positive. The numerical example in Appendix A illustrates these.

We also consider the special case of $\frac{P(Q)}{B^*(Q)}|_{Q=0}$. In this case, B^* solves $\int_{B^*}^{I}(I-B)f(B)\,dB=\int_0^{B^*}Bf(B)\,dB$ by noting that B^* has a unique solution based on (2), and $\Gamma=\frac{\pi C\cdot F(B^*)+X}{1-\pi}$. Hence, r is determined. We can also work out $\frac{dr}{dQ}$ at $Q=0$ by (A9). That is, $\frac{dr}{dQ}|_{Q=0}=\frac{CB^*-[\pi C\cdot F(B^*)+X]\left(\frac{I}{1-\pi}+\frac{1}{\pi f(B^*)}\right)}{\left(\frac{1-\pi}{\pi}B^*+Q\right)B^*I}$. *Ceteris paribus*, if π is sufficiently low, $\frac{dr}{dQ}|_{Q=0}<0$.

To summarize, when $X>\underline{X}$, we can choose some function $f(B)$ (i.e., $f(B)$ is sufficiently high in some region of B) and some $\overline{Q}$ such that r decreases first and then increases in Q within the interval $Q\in\left[0,\overline{Q}\right]$.

Proof of Proposition 5: First, given a Q_i for each sector, we solve for the equilibrium within each sector, that is, we solve for the triplet $\{B_i^*,P_i,r_i\}$. In particular, we obtain the function $r_i(Q_i)$. If conditions in Proposition 4 are satisfied, $r_1(Q_1)$ is a decreasing function and $r_2(Q_2)$ is a 'U'-shaped function. Second, by considering the link between the two sectors, (5a) and (5e), we can work out Q_i (for $i=1$ and 2) for a given Q. That is, by considering $r_1(Q_1)=r_2(Q_2)=r$ and $Q_1+Q_2=Q$, we obtain the unique Q_1 and Q_2, and r. In fact, by aggregating $r_1(Q_1)$ and $r_2(Q_2)$, we can obtain an 'aggregate' function $r(Q)$. That is, for a given r, we find the corresponding Q_1 which solves $r_1(Q_1)=r$ and Q_2 which solves $r_2(Q_2)=r$, and then aggregate Q_1 and Q_2 as $Q=Q_1+Q_2$. Under a wide set of chosen parameters, the 'aggregate' function $r(Q)$ is a 'U'-shaped function. The numerical example above in Appendix A is one case. Thus, for a given Q, we have unique Q_1, Q_2 and r.

Proof of Proposition 6: From the proof of Proposition 5, we know that the 'aggregate' function $r(Q)$ is a 'U'-shaped function (i.e., decreasing first and then increasing). By Proposition 4, $r_1(Q_1)$

is a decreasing function. Therefore, to maximize Q_1, we need to choose a Q to minimize $r(Q)$. Clearly, there is a unique Q that minimizes r and thus maximizes Q_1.

Proof of Proposition 7: Considering that $r_1(Q_1)$ is a decreasing function and $r_2(Q_2)$ is a 'U'-shaped function, we find a unique Q_1^* that solves $r_1(Q_1^*) = r_{\min}$ and a unique Q_2^* that solves $r_2(Q_2^*) = r_{\min}$. We define $Q^* = Q_1^* + Q_2^*$. By $r_1(Q_1) = r_2(Q_2) = r$ and $Q_1 + Q_2 = Q$, we have that Q_1 is increasing in Q when $Q < Q^*$ and decreasing in Q when $Q > Q^*$, and that Q_2 is increasing in Q.

Proof of Proposition 8: Given Q_i and C_i for each sector, solve for the equilibrium within each sector, that is, solve for the triplet $\{B_i^*, P_i, r_i\}$. Based on the proofs of Propositions 3 and 4, the cash flow C affects the results in Propositions 3 and 4 only quantitatively, not qualitatively. Therefore, for some C_1 and C_2, $P_1(Q_1)$ and $P_2(Q_2)$ have the properties in Proposition 3, and $r_1(Q_1)$ and $r_2(Q_2)$ have the properties in Proposition 4.

In addition, we need that the asset price in Sector 1 is binding at $E^L(\widetilde{x})$ and that in Sector 2 is not. Note that $C_1 > C_2$ and $X_1 < X_2$. When π is chosen to be sufficiently small, the first term in the numerator of Γ in (3) which involves C has a limited role in determining the asset price while the second term which is about X becomes crucial. Hence, when the gap between X_1 and X_2 is sufficiently big, the result is obtained.

Overall, the conditions to guarantee that Program 1 has a unique interior optimal Q $(\in (0, \overline{Q}))$ are that π is sufficiently small and $f(B)$ is sufficiently high in some region of B.

A numerical example is given to illustrate the existence of relevant parameters. We continue the previous simulation exercise in Appendix A. Let $I = 1$, $\pi = 0.185$, $\mathbb{E}^H(\widetilde{x}) = 1.2$, and $\mathbb{E}^L(\widetilde{x}) = 1.1$. The distribution of B is $f(B) = \log \frac{1}{1-B}$ for $B \in [0, 1)$. As the maximum total amount of liquidity that any one sector can demand is $Q^{\max} = \int_0^1 Bf(B)dB = 0.75$, we set $\overline{Q} = 2Q^{\max} = 1.5$. We choose $C_1 = 5.6 > C_2 = 4$ and $X_1 = 0.01 < X_2 = 0.4$. We find that $P_1(Q_1) = 1.1$ for any $Q_1 \in [0, 0.75]$ and that when Q_2 is sufficiently high $P_2(Q_2)$ is above 1.1 and increasing in Q_2. The optimal level of liquidity injection to maximize Q_1 is $Q^* = 1.1787$, which lies within $(0, \overline{Q})$, where $\overline{Q} = 1.5$.

Proof of Proposition 10: First, we show that W is divided among investing entrepreneurs, non-investing entrepreneurs, and the government. For simplicity, we first consider the one-sector economy. By using (2), we have

$$\begin{aligned} W &= \int_0^{B^*} (C-I) f(B)\, dB \\ &= \underbrace{\int_0^{B^*} [C-(1+r)B-(I-B)] f(B)\, dB}_{\text{Surplus for investing firms}} + \underbrace{r\int_{B^*}^{I} (I-B) f(B) dB}_{\text{Surplus for non-investing firms}} + \underbrace{rQ}_{\text{Surplus for the government}} . \end{aligned}$$

For the two-sector economy, it is easy to show:

$$\begin{aligned} W &= \sum_i \int_0^{B_i^*} (C_i-I) f(B)\, dB \\ &= \underbrace{\sum_i \int_0^{B_i^*} [C_i-(I-B)-(1+r)B] f(B)\, dB}_{\text{Surplus for investing firms}} + \underbrace{\sum_i r\int_{B_i^*}^{I} (I-B) f(B) dB}_{\text{Surplus for non-investing firms}} + \underbrace{rQ}_{\text{Surplus for the government}} . \end{aligned}$$

Second, by using (2), it is easy to show that W can be rewritten as

$$W = \left[Q + 2\int_0^I (I-B) f(B) dB\right] \cdot \bar{R} - 2\int_0^I (I-B) f(B) dB - Q.$$

The first-order condition of W with respect to Q is

$$\bar{R}(Q) + \left[Q + 2\int_0^I (I-B) f(B) dB\right] \frac{d\bar{R}(Q)}{dQ} = 1. \tag{A10}$$

Clearly, $\bar{R}(Q)$ is decreasing in Q for $Q > Q^*$ due to the crowding-out effect (i.e., $F(B_1^*)$ is decreasing and $F(B_2^*)$ is increasing in Q). Under some parameter values, the LHS of (A10) overall is a decreasing function of Q for some range of Q, and there is a unique solution to (A10).

To obtain further insight, we can rewrite W as

$$\begin{aligned} W = &\left[\underbrace{C_2 \cdot \begin{pmatrix} F(B_1^*(Q_1)) + F(B_2^*(Q_2)) \\ -F(B_1^*(Q_1^*)) - F(B_2^*(Q_2^*)) \end{pmatrix}}_{\text{additional injected liquidity entering Sector 2}} - \underbrace{(C_1 - C_2) \cdot (F(B_1^*(Q_1^*)) - F(B_1^*(Q_1)))}_{\text{some liquidity moving out of Sector 1 to Sector 2}} \right] \\ &+ \begin{bmatrix} C_1 \cdot F(B_1^*(Q_1^*)) \\ +C_2 \cdot F(B_2^*(Q_2^*)) \end{bmatrix} - 2\int_0^I (I-B) f(B) dB - Q, \end{aligned}$$

where Q_i^* is the net liquidity inflow for Sector i when $Q = Q^*$ in Proposition 8. By $2\int_0^I (I - B) f(B) dB + Q = I \cdot (F(B_1^*) + F(B_2^*))$, we have that $F(B_1^*(Q_1)) + F(B_2^*(Q_2))$ is increasing in Q. Also, Q_1 and hence $F(B_1^*(Q_1))$ are decreasing in Q. That is, an increase in Q has two opposite forces on income at T_1: additional liquidity injected entering Sector 2 increases income, but more liquidity injected also crowds more liquidity out from Sector 1 to Sector 2, decreasing income because $C_1 > C_2$. Under certain conditions, the second force dominates the first force when Q is too high, and, therefore, there exists an optimal Q.

We continue the numerical example in the proof of Proposition 8. The optimal level of liquidity injection that maximizes W is coincidentally also $Q = 1.1787$ in Proposition 10. That is, any additional liquidity injection beyond Q^* (the optimal level that maximizes B_1^*) will reduce the total surplus.

Proof of Lemma 11: Denote by α the proportion of entrepreneurs from Sector 1 who decide to trade in asset market 2. We now explore three alternative scenarios regarding agents' beliefs, under which we draw the same conclusion.

1) Beliefs of an entrepreneur are perfectly correlated in the two asset markets.

In this scenario, an entrepreneur who has high beliefs in asset market 1 necessarily also has high beliefs in asset market 2, so $\alpha = 0$. This is because from the perspective of high-beliefs entrepreneurs, asset market 1 is more undervalued and thus has a higher return to speculation than asset market 2 (i.e., $\frac{\mathbb{E}^H(\widetilde{x})}{P_1} > \frac{\mathbb{E}^H(\widetilde{x})}{P_2}$ by $P_1 < P_2$). So high-beliefs speculators in Sector 1 stay and focus on trading in Sector 1, rather than migrate and divert funds to trade in asset market 2. Because $\alpha = 0$, the results in the main model do not change.

2) Beliefs of an entrepreneur are not perfectly correlated in the two asset markets.

In this scenario, $\alpha > 0$. Entrepreneurs from Sector 1 with low beliefs in market 1 but high beliefs in market 2 participate in trading in market 2. Then, the asset price Γ_1 of Sector 1, given in (3), remains the same. This is because neither the sellers nor the buyers in market 1 change, by noting that those who migrate to trade in Sector 2 must be among low-beliefs sellers in market 1.

However, the asset price Γ_2 of Sector 2 changes and becomes

$$\Gamma_2 = \frac{\pi \left[C_2 \cdot F\left(B_2^*\right) - Q_2\left(1+r\right)\right] + \alpha \left\{\left[C_1 \cdot F\left(B_1^*\right) - Q_1\left(1+r\right)\right] + P_1\right\} + X_2}{1-\pi}. \tag{A11}$$

Compared with Γ in (3), there is an extra term in the numerator of Γ_2 in (A11): entrepreneurs from Sector 1 use their cash income from projects or deposits (i.e., the term $C \cdot F\left(B_1^*\right) - Q_1\left(1+r\right)$) as well as their income from selling assets in market 1 (i.e., the term P_1) to purchase assets in market 2. We can prove that Γ_2 in (A11) still has the same properties as in Propositions 3 and 4, and hence the conclusions of the model do not change. Concretely, we first show that Γ_2 is increasing in Q. We discuss two ranges of Q: $\frac{dr}{dQ} < 0$ (the range of the allocation effect) and $\frac{dr}{dQ} > 0$ (the range of the crowing-out effect). For the first range, both Q_1 and Q_2 are increasing in Q. By the proof of Lemma 2, under the sufficient condition of $C_i - I(1+r) \geq 0$, we have that $C_i \cdot F\left(B_i^*\right) - Q_i\left(1+r\right)$ is increasing in Q. Also, P_1 is a constant with $P_1 = \mathbb{E}^L(\widetilde{x})$. Hence, Γ_2 is increasing in Q. For the second range, because $r = \frac{\Gamma_2}{B_2^*} - 1$ and B_2^* is increasing in Q, we have that Γ_2 is increasing in Q. For the curve of $r_2(Q_2)$ to be $\cup$-shaped, as shown in the Proof of Proposition 4, $f(B)$ needs to be sufficiently high in some region of B.

3) There exist no heterogeneous beliefs among entrepreneurs from Sector 1.

In this scenario, we keep the setup for Sector 2 the same as in the main model but assume a slightly different setup for Sector 1. It is alternatively assumed that entrepreneurs do not have heterogeneous beliefs about a project in Sector 1. In other words, a project's valuation at T_1 is common knowledge among entrepreneurs for Sector 1. This setup essentially implies that there is *no speculative trade* in Sector 1 while there is in Sector 2; in other words, entrepreneurs have no disagreement regarding a project's valuation (i.e., on cash flow $\widetilde{x}$) at T_1 in the real sector.

To model this setup without introducing new notations, simply set $\pi = 0$ for Sector 1; that is, all entrepreneurs in Sector 1 have common beliefs: $\Pr[\widetilde{x} = u] = \theta_L$.[1] Then, it is easy to show that P_1 is a constant and does not change with the liquidity injection, i.e., $P_1 = \mathbb{E}^L(\widetilde{x})$.[2] As in scenario 2), entrepreneurs of Sector 1 may have high beliefs in market 2, and so they participate in buying assets in market 2. That is, the asset price Γ_2 of Sector 2 is given by (A11). Then, it is easy to show that the model results under scenario 3) are the same as those under scenario 2).

Proof in Section 5.3: It is easy to obtain the result based on Figure 4. When Q is low such that $r_2(Q_2)$ is decreasing in Q_2, r_2 is decreasing in Q; by $r_1 - r_2 = \Delta r$, in equilibrium r_1 is also decreasing in Q, which means that Q_1 is increasing in Q. When Q is high such that $r_2(Q_2)$ is increasing in Q_2, r_2 is increasing in Q; by $r_1 - r_2 = \Delta r$, in equilibrium r_1 is also increasing in Q, which means that Q_1 is decreasing in Q. In short, the model results (Propositions 5-8 and 10) do not change qualitatively under the new assumption that $r_1 - r_2 = \Delta r$.

Section 4.1: Crowding-out effect with a falling interest rate One implication of the main model is that in equilibrium the crowding-out effect is accompanied by an increase in the interest rate. However, our model framework also shows that the crowding-out effect can be accompanied by a decrease in the interest rate. In fact, when $r_1(Q_1)$ is increasing in Q_1 (in some region of Q_1) and $r_2(Q_2)$ is decreasing in Q_2 (in some region of Q_2), such an equilibrium outcome can occur. Figure B1 illustrates such an equilibrium. In the figure, when an additional $\Delta Q = (Q_1' + Q_2') - (Q_1 + Q_2)$ of liquidity is injected, Sector 2 enters a liquidity-asset price 'inflationary' cycle while Sector 1 enters a 'deflationary' cycle, and the two cycles reinforce each other. Intuitively, when Sector 1 loses some liquidity, its asset price will quickly fall, causing its affordable interest rate to go down.

[1] To save notations, the probability under the common belief is written as θ_L.

[2] For simplicity, we assume that α is small, so that the cash-in-the-market price in market 1, induced by the selling from those "liquidity" traders who have "better" investment opportunities in the other market, will not drop below the asset's fundamental value.

Because of the higher price (interest rate) elasticity of demand to liquidity for Sector 2, at the lower interest rate Sector 2 is able to absorb and accommodate more liquidity than the initial lost amount of Sector 1 and thus will necessarily suck additional liquidity out of Sector 1.

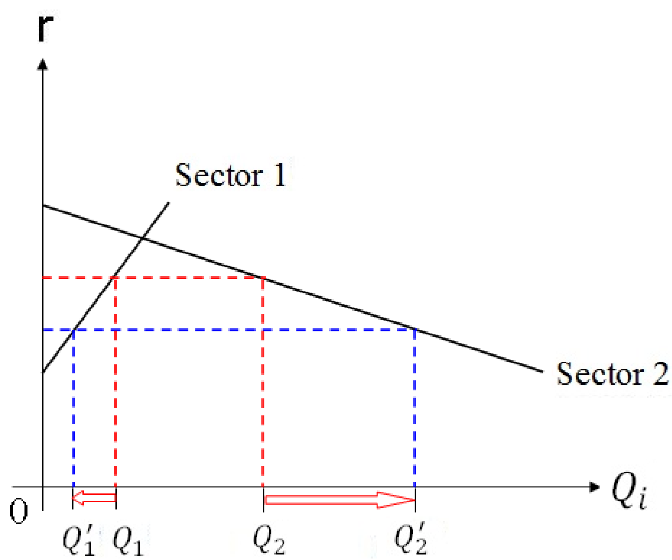


Figure B1: Crowding-out effect accompanied by a decrease in the interest rate

We construct the equilibrium in which $r_1(Q_1)$ is increasing in Q_1 (in some region of Q_1) and $r_2(Q_2)$ is decreasing in Q_2 (in some region of Q_2). The following is one case. By $1+r=\frac{P(Q)}{B^*(Q)}$, if P increases faster than B^*, r is increasing in Q; otherwise, r is decreasing in Q. For Sector 2, if X_2 is sufficiently high such that Γ_2 (i.e., Γ in (3) for Sector 2) is binding at the upper bound $\mathbb{E}^H(\widetilde{x})$, then clearly $r_2(Q_2)$ is decreasing in Q_2. For Sector 1, if X_1 is sufficiently lower such that Γ_1 (i.e., Γ in (3) for Sector 1) is lower than and not binding at $\mathbb{E}^H(\widetilde{x})$, then based on the proof of Proposition 4, we can choose some function $f(B)$ (i.e., $f(B)$ is sufficiently high in some region of B) such that $r_1(Q_1)$ is increasing in Q_1. Furthermore, we need that the 'aggregate' function $r(Q)$ is decreasing in Q, where $r(Q)$ is defined in the proof of Proposition 5; that is, the price (interest rate) elasticity of demand to liquidity for Sector 2 is higher than that for Sector 1. To achieve this, it is realistic to assume the two sectors are heterogeneous in cash flow C and asset specificity X as well as in cash flow $\widetilde{x}$ and distribution $f(B)$.

A numerical example is to illustrate the above equilibrium. Let $I=1$ and $\pi=0.4$. For Sector 1, the distribution of B is a truncated normal with $f(B)=\frac{100}{\Phi(0.508)-\Phi(0)}\varphi\left(\frac{B-0.5}{0.01}\right)$ in the support $[0,0.508]$, where $\varphi(\cdot)$ stands for the p.d.f. of the standard normal, and $C_1=1.8$, $X_1=0.15$, $\mathbb{E}^H(\widetilde{x}_1)>0.8781$, and $\mathbb{E}^L(\widetilde{x}_1)<0.8544$, where $\widetilde{x}_i$ denotes the stochastic cash flow $\widetilde{x}$ at T_2 for Sector $i=1$ and 2. For Sector 2, the distribution of B is uniform within the support $[0.5007, 0.5037]$, and $C_2=1.795$, $X_2=0.163$, and $\mathbb{E}^L(\widetilde{x}_2)<\mathbb{E}^H(\widetilde{x}_2)=0.8671$. Denote by $Q_i^{\max}$ the maximum liquidity demand for Sector $i=1$ and 2. Then, we find out that $P_1(Q_1)$ is monotonically increasing in $Q_1 \in [0, Q_1^{\max}]$ with $Q_1^{\max} = 0.4963$, while $P_2(Q_2)$ is binding at its asset price upper bound $\mathbb{E}^H(\widetilde{x}_2) = 0.8671$ for all $Q_2 \in [0, Q_2^{\max}]$ with $Q_2^{\max} = 0.5022$. Figure B2 depicts $r_1(Q_1)$ and $r_2(Q_2)$.

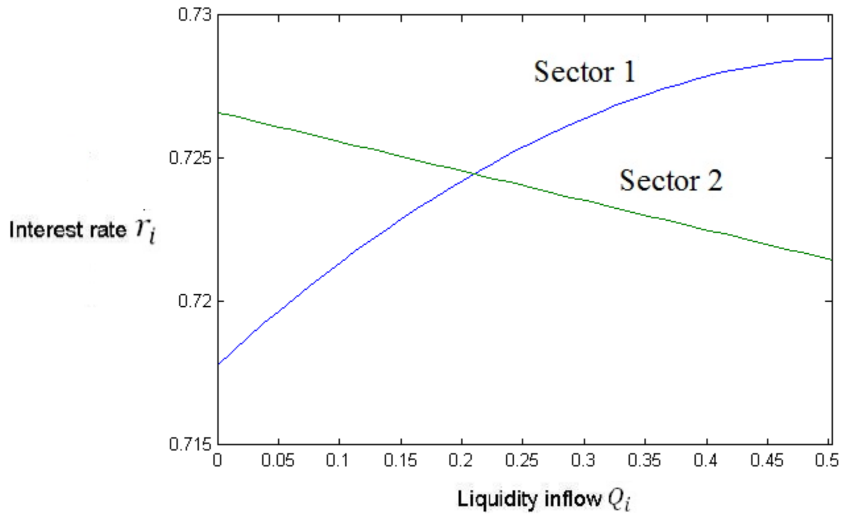


Figure B2: A numerical example of crowding-out with a falling interest rate